Behavior Analysis of a System with Ecological Failure and Preventive Resume Repair.

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Abstract: *This Problem is related to the Reliability analysis of a system with ecological failures and preventive resume repair policy. The system under consideration is Non-Markovian; supplementary variables have been used to convert this in Markovian. Mathematical model of considered system has been solved be using Laplace transform. Transition-state diagram has been shown in fig-1(b). Reliability, M.T.T.F and availability of the system have been computed. Ergodic behavior for the considered system has been obtained to improve practical utility of the model. A numerical illustration and its graphical representation have also been appended in the end to highlight important results of the study.*

Keywords: Markovian System, Supplementary Variables, Reliability, M.T.T.F. etc

1. INTRODUCTION

We all have some idea of the word Reliability. We often hear and make remark to the effect that a particular brand of electronic mixer, or motor, or refrigerator or an automobile is Reliable or un reliable. When we say that an air conditioner manufactured by a particular firm is Reliable then we mean its performance is trouble free: but we do not completely rule out the possibility of failure. However, based on the quality of the components used in the manufacture of that air conditioner and our experience with other unit of the same firm, we conclude that the chance of its failure is few.

In other words a certain degree of uncertainty about the performance of equipment involved. The degree of uncertainty and certainty is related to the amount of the information available about the particular equipment like: about its design characteristic, The quality of the component used, its performance, etc. The combined knowledge of these factors gives us a descriptive picture of the word "Reliability".

Fundamentals:-

Every science is based on some fundamental concepts and definitions; same is true for the theory of reliability. Some of the basic concepts on which the theory of reliability is formulated are given below.

System: A system is an arbitrary device consisting of different parts or components or units.
Time: It is the period during which one can expect the satisfactory performance of a system.
Adequate: It indicates the criteria for operations of the device to be satisfactory.
Failure mode: It is the effect by which a failure is observed. There are many different types of failure modes like:
Catastrophic failures and Degradation failures.

Failure Rate: The number of failures in a specified time period.
Up time: It is the total time during which the system is inacceptable operating conditions.
Down time: It is the time during which the system is not in acceptable operating conditions.
Mean time to failure: MTTF is the mean operating time between successive failures.
Maintainability:

"The probability that a device that has failed will be restored to operational effectiveness within a given period of time when the maintenance action is performed in accordance with prescribed procedures. "

Availability: is the measure of the ability of an item to complete a mission without excessive down time.

System Environment: Characteristic of the environment in which the system operates, including natural factors (e.g., flood lightning, earthquake, fire, tornado etc.) Ecological stress or shocks, man made hazards, and shared infrastructure can also generate Ecological failures.

Stand By Redundancy

The standby redundancy represents a situation with one unit operating and a number of on standby. In general, there are three types in standby, i.e. cold, hot and warm standby. Cold standby implies that the inactive components have a zero failure rate and cannot fail while in standby state. Hot standby implies that an inactive component has the same failure rate as when it is in operation. Warm standby is an intermediate case and it implies that an inactive component has a failure rate between that for the cold and hot
Stand by. It is also called dormant failure in some papers.

Reliability

"Reliability is the probability that a device, a system, or Process will perform its prescribed duty without failure for a given time when operated correctly in a Specified environment.**"** This complicated reliability definition involves:

- Probability (chances) for survival [absence of absolute statements about survival]
- Components, systems, and processes [the issue is complex--not simple]
- functioning without failure [reliability (survival) terminates in a failure (death)]
- function for a given time period [state the elapsed time expected--nothing survives forever]
- Duty cycles under the concept of prescribed duty [design for the cycle peaks—not the "average"]
- Correct operation is a key element for survival [you must know right from wrong]
- Design for use in the correct environment [successes in one environment is failure for another].

Mean time to failure: MTTF is the mean operating time between successive failures.

System Description:

In this model, the whole system has divided in two subsystems namely, A and B, connected in series. The subsystem A has three units in series. On failure of any one unit of subsystem A, the whole System goes to failed state. The subsystem B consists of three units connected in parallel redundancy. The Subsystem B is of 2-out-of-3: F and 1-out-of-3: D nature. The whole system can also fail due to Ecological reasons. All failures follow exponential time distribution whereas all repairs follow general time distribution. Preventive resume policy has been adopted for repair purpose. The subsystem A has given priority in repairs over the subsystem B.

Assumptions

The following assumptions have been taken care in this study:
(1) Initially all the units are in operable condition of full efficiency.
(2) Failures are S-independent and nothing can fail from a failed state.
(3) All failures follow exponential time distribution.
(4) Repairs follow general time distribution. Repair facilities are always available and are perfect.
(5) Preventive resume policy has been adopted for repair purpose. The subsystem A has given Priority in repairs over the system B.
(6) The subsystem B is of 1-out-of-3:D.
(7) The whole system can also fail due to Ecological reasons.
(6) The subsystem B is of 1-out-of-3:D and 2-out-of-3:F.
(7) The whole system can also fail due to Ecological reasons

Notation

λi (i = 1,2,3) = Failure rate of *i th* unit of subsystem A.

$\lambda = \sum \lambda_i \ (i=1, 2, 3)$

$\mu 1$ / μ 2 = Failure rate of first/second unit of subsystem B.

$e1, e2$ = Failure rates due to Ecological reasons.

$\alpha_1 (x)\Delta$ / $\alpha_2 (m)\Delta$= The Probability that subsystem A /Ecological failure will be repaired in the time interval $(x, x+\Delta)$ /$(m, m+\Delta)$, conditioned that it was not repaired up to the time x/m.

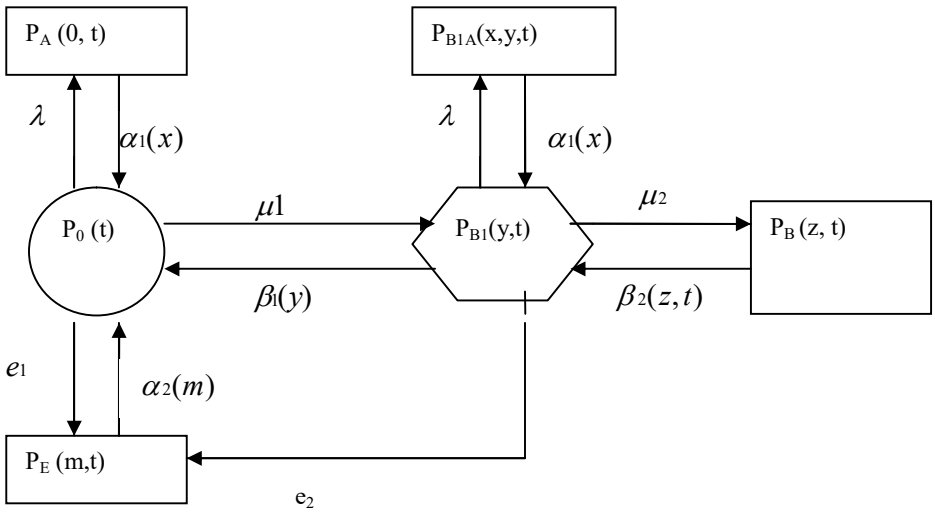
$\beta_1 (y)\Delta$ / $\beta_2 (z)\Delta$ = The probability that one/two units of subsystem B will be repaired in the time interval $(y, y+\Delta)$ /$(z, z+\Delta)$, conditioned that it was not repaired up to the time y/z.

$P_0 (t)$ = Pr {at time t, system is all operable}.

$P_i (j, t)\Delta$ = Pr {at time t, system suffers with *i th* failure}. Elapsed repair time lies in the interval $(j, j+\Delta)$.
$P_{B1} (y, t)\Delta$ = at time t, system works with reduced efficiency due to failure of any one unit of subsystem B}. Elapsed repair time lies in the interval (y, $y+\Delta$).

$P(s)$ = *Laplace transform (L.T) of function P (t).*

M .T .T .F. = Mean time to failure.
$B_1A\ (x, y, t)\ \Delta$ = Pr {at time t, system is failed due to failure of subsystem A while one unit of subsystem B has already failed}. Elapsed repair time for A lies in the interval $(x, x+\Delta)$ and for B it lies within $(y, y+\Delta)$.



State Transition Diagram.(Figure 1)

Formulation of the mathematical Model: The set of differential equations,
Which governing the behavior of the considered system is:

$$\left[\frac{d}{dt}+\lambda+\mu_1+e_1\right]P_0(t)=\int P_A(x,t)\alpha_1(x)dx+\int P_E(m,t)\alpha_2(m)dm+\int P_{B1}(y,t)\beta_1(y)dy$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial t}+\alpha_1(x)\right]P_A(x,t)=0 \qquad (2)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\lambda+\mu_2+e_2+\beta_1(y)\right]P_{B1}(y,t)=\int_0^{\infty}P_{B1A}(x,y,t)\alpha_1(x)dx \quad (3)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\alpha_1(x)\right]P_{B1A}(x,y,t)=0 \quad (4)$$

$$\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial t}+\beta_2(z)\right]P_B(z,t)=0 \quad (5)$$

$$\left[\frac{\partial}{\partial m}+\frac{\partial}{\partial t}+\alpha_2(m)\right]P_E(m,t)=0 \quad (6)$$

Boundary conditions are:

$$P_A(0,t)=\lambda P_0(t) \quad (7)$$

$$PB_1(0,t)=\mu_1 P_0(t)+\int_0^{\infty}PB(z,t)\beta_2(z)dz \quad (8)$$

$$P_{B_1A}(0,y.t)=\lambda P_{B_1}(y,t) \quad (9)$$

$$P_B(0,t)=\mu_2 P_{B_1}(t) \quad (10)$$

$$P_E(0,t)=e_1P_0(t)+e_2P_{B_1}(t) \quad (11)$$

Initial Conditions:

$P_0(0) = 1$, otherwise 0. (12)

Solution of the Model:

In order to solve the above Mathematical model, trans-state probabilities of the model are computed. We shall use Laplace transform to solve the above mathematical model. Taking laplace transforms of equations from (1) to (6), Subjected to initial condition and then solving them, we obtain:

$$\overline{P_0(s)}=\frac{1}{C(s)} \quad (13)$$

$$\overline{P_A(s)}=\frac{\lambda D_{\alpha 1}(s)}{C(s)} \quad (14)$$

$$\overline{P_{B1}(s)}=\frac{B(s)}{C(s)} \quad (15)$$

$$\overline{P}_{B1A}(s)=\frac{\lambda D_{\alpha 1}(s)}{C(s)}\left[\mu_1+\mu_2 B(s)\bar{s}_{\beta 2}(S)\right]D_{\beta 1}(A) \quad (16)$$

$$\overline{P}_B(s)=\frac{\mu_2 B(s)}{C(s)}\left[e_1+e_2 B(s)\right] \quad (17)$$

$$\overline{P}_B(s)=\frac{\mu_2 B(s)}{C(s)}D_{\beta 2}(s) \quad (18)$$

Where,

$$A=s+\lambda+\mu 2+e2-\lambda\bar{S}\alpha 1(s) \quad (19)$$

$$B(s) = \frac{\mu_1 D_{\beta 1}(A)}{1 - \mu_2 \overline{S}_{\beta 2}(s) D_{\beta 1}(A)} \tag{20}$$

$$C(s) = s + \lambda + \mu_1 + e_1 - \lambda \overline{S\alpha_1(s)} - \left[e_1 + e_2 B(s)\right] \overline{S_{\alpha 2}(s)} - \left[\mu_1 + \mu_2 B(s) \overline{S_{\beta 2}(s)}^{-}\right] \overline{S_{\beta 2}(A)} \tag{21}$$

Asymptotic Behavior

Using final value theorem of Laplace Transform i.e. $\lim P(t) = Lims\overline{P}(s) = P$,

We obtain the following ergodic behavior of the considered system:

$$P_0 = \frac{1}{C'(0)} \tag{22}$$

$$P_A = \frac{\lambda M_{\alpha 1}}{C'(0)} \tag{23}$$

$$P_{B1} = \frac{B(0)}{C'(0)} \tag{24}$$

$$P_{B1A} = \frac{\lambda M \alpha_1}{C'(0)} \left[\mu_1 + \mu_2 B(0)\right] D_{\beta 1}(\mu_2 + e_2) \tag{25}$$

$$P_B = \frac{\mu_2 B(0)}{C'(0)} M_{\beta 2} \tag{26}$$

$$P_E = \frac{M_{\alpha 2}}{C'(0)} \left[e_1 + e_2 B(0)\right] \tag{27}$$

Where,

$$C'(0) = \left[\frac{d}{ds} C(s)\right] \tag{28}$$

$$Mi = -\overline{S'i}(0) = \tag{29}$$

$$B(0) = \frac{\mu 1 D \beta 1(\mu 2 + e2)}{1 - \mu 2 D \beta 1(\mu 2 + e2)} \tag{30}$$

Verification:

$P_{up}(t) + P_{down}(t) = 1/s$

Reliability and M.T.T.F. of the System: The Laplace transform of the reliability when all repair rates are zero of the system is given by:

$$\overline{R(s)} = \frac{1}{s + \lambda + \mu_{1+} e_1}$$

Taking inverse L.T., we get,

$$R(t) = \exp.\{-(\lambda + \mu_1 + e_1)t\} \tag{31}$$

Also, $MTTF = \lim \overline{R(s)}$ (As s tends to 0)

$$= \frac{1}{\lambda + \mu_1 + e_1} \tag{32}$$

Results and Discussion

Figure 2 shows the values of "Reliability Vs. Time" and Analysis of that yields that reliability of system decreases approximately in a constant manner. The values of "M.T.T.F. Vs. λ "have given in figure 3. Examination of this reveals that M.T.T.F. of considered system decreases as we make

increases in the value of failure rate λ of subsystem, It should be noted that there are no sudden jumps in the values of R(t), M.T.T.F.

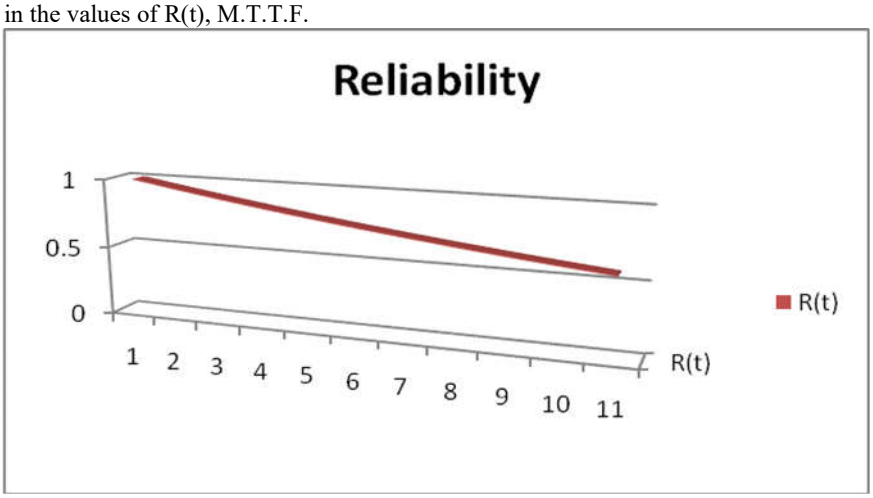


Figure 2 Reliabilty

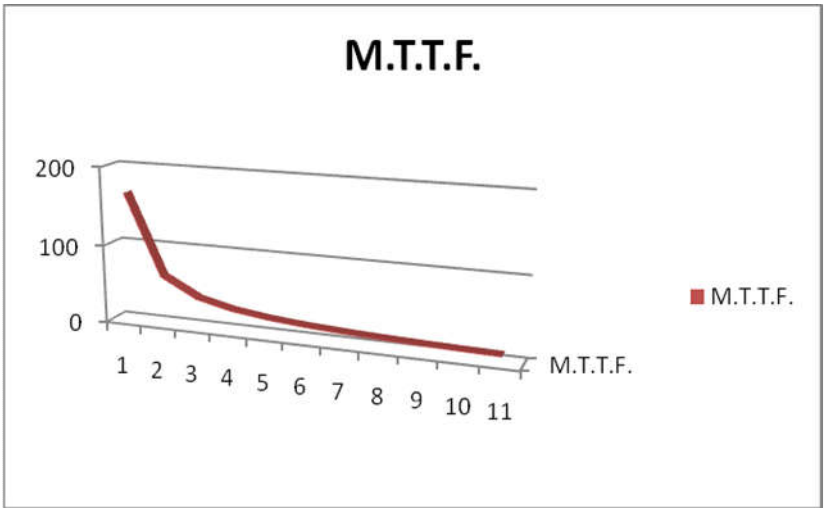


Figure 3 MTTF

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