Fuzzy Quotient -3 Cordial Labeling of Some Trees of Diameter 2, 3 and 4

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Abstract: *Let $G(V,E)$ be a simple, finite, undirected and planar graph of order p and size q. Let $\sigma: V(G) \to [0,1]$ be a function defined by $\sigma(v) = \frac{r}{10}$, $r \in Z_4 - \{0\}$. For each edge uv define $\mu: E(G) \to [0,1]$ $by\ \mu(uv) = \frac{1}{10}\left\lceil\frac{3\sigma(u)}{\sigma(v)}\right\rceil$ where $\sigma(u) \leq \sigma(v)$. The function σ is called fuzzy quotient-3 cordial labeling of G if the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1, the number of edges labeled with i and the number of edges labeled with j differ by at most 1 where $i,j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \neq j$. The number of vertices having label i denotes $v_\sigma(i)$and the number of edges having label i denotes $e_\mu(i)$. Here it is proved that some trees of diameter 2, 3 and 4 are fuzzy quotient -3 Cordial.*

Key words: Tree, diameter of tree and Fuzzy quotient-3 cordial graph.

1. INTRODUCTION

In 1967, Rosa introduced the concept of labeling in graph theory. Graph labeling is an assignment of integers to the vertices or edges or both under certain conditions. Graph labeling is used in several areas of science and technology like coding theory, astronomy, circuit design etc. The cordial labeling concept was first introduced by cahit [2]. We introduced fuzzy quotient -3 cordial labeling of graphs and proved some star related graphs are fuzzy quotient -3 cordial labeling [3], [4].

Let $\sigma: V(G) \to [0,1]$ be a function defined by $\sigma(v) = \frac{r}{10}$, $r \in Z_4 - \{0\}$. For each edge uv define $\mu: E(G) \to [0,1]$ $by\mu(uv) = \frac{1}{10}\left\lceil\frac{3\sigma(u)}{\sigma(v)}\right\rceil$ where $\sigma(u) \leq \sigma(v)$. The function σ is called fuzzy quotient-3 cordial labeling of G if the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1, the number of edges labeled with i and the number of edges labeled with j differ by at most 1 where $i,j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \neq j$. The number of vertices having label i denotes $v_\sigma(i)$and the number of edges having label i denotes $e_\mu(i)$. In this paper we proved that some trees of diameter 2, 3 and 4 are fuzzy quotient -3 Cordial.

2. NOTATIONS

Notation 2.1:
Let ${\mathcal{T}_1}^2$ be the tree diameter 2 obtained by attaching n pendant edges to the internal vertex of the path P_3.

Notation 2.2:
The trees of diameter 3 denoted by ${\mathcal{T}_s}^3$, $1 \leq s \leq 4$ are defined as follows.

1. Let $\mathcal{T}_1^{\,3}$ be the tree obtained by attaching n pendant edges to the first internal vertex of the path P_4.
2. Let $\mathcal{T}_2^{\,3}$ be the tree obtained by attaching n pendant edges to the second internal vertex of the path P_4.
3. Let $\mathcal{T}_3^{\,3}$ be the tree obtained by attaching m, n pendant edges to the first and the second internal vertices of the path P_4.
4. Let $\mathcal{T}_4^{\,3}$ be the tree obtained by attaching n pendant edges through an edge to the internal vertex of the path P_3.

Remarks 2.3:
By the above notations $\mathcal{T}_1^{\,3} \cong \mathcal{T}_2^{\,3}$.

Notation 2.4:
The trees of diameter 4 denoted by $\mathcal{T}_s^{\,4}$, $1 \leq s \leq 22$ are defined as follows.

1. Let $\mathcal{T}_1^{\,4}$ be the tree obtained by attaching n pendant edges to the first internal vertex of the path P_5.
2. Let $\mathcal{T}_2^{\,4}$ be the tree obtained by attaching n pendant edges to the second internal vertex of the path P_5.
3. Let $\mathcal{T}_3^{\,4}$ be the tree obtained by attaching n pendant edges to the third internal vertex of the path P_5.
4. Let $\mathcal{T}_4^{\,4}$ be the tree obtained by attaching m, n pendant edges to the first and second internal vertices of the path P_5.
5. Let $\mathcal{T}_5^{\,4}$ be the tree obtained by attaching m, n pendant edges to the first and third internal vertices of the path P_5.
6. Let $\mathcal{T}_6^{\,4}$ be the tree obtained by attaching m, n pendant edges to the second and third internal vertex of the path P_5.
7. Let $\mathcal{T}_7^{\,4}$ be the tree obtained by attaching m, n and l pendant edges to the internal vertices of the path P_5.
8. Let $\mathcal{T}_8^{\,4}$ be the tree obtained by attaching n pendant edges through an edge to the first internal vertex of the path P_4.
9. Let $\mathcal{T}_9^{\,4}$ be the tree obtained by attaching n pendant edges through an edge to the second internal vertex of the path P_4.
10. Let $\mathcal{T}_{10}^{\,4}$ be the tree obtained by attaching n pendant edges through an edge to the middle vertex of the path P_5.
11. Let $\mathcal{T}_{11}^{\,4}$ be the tree obtained by attaching n path of length two to the internal vertex of the path P_3.
12. Let $\mathcal{T}_{12}^{\,4}$ be the tree obtained by attaching n path of length two to the first internal vertex of the path P_4.

13. Let ${\mathcal{T}_{13}}^4$ be the tree obtained by attaching n path of length two to the second internal vertex of the path P_4.

14. Let ${\mathcal{T}_{14}}^4$ be the tree obtained by attaching n path of length two to the middle vertex of the path P_5.

15. Let ${\mathcal{T}_{15}}^4$ be the tree obtained by attaching n path of length two through an edge to the internal vertex of the path P_3.

16. Let ${\mathcal{T}_{16}}^4$ be the tree obtained by attaching n pendant edges through the path of length two to the internal vertex of the path P_3.

17. Let ${\mathcal{T}_{17}}^4$ be the tree obtained by attaching m pendant edges to the first internal vertex and attaching n pendant edges through an edge to the second internal vertex of the path P_4.

18. Let ${\mathcal{T}_{18}}^4$ be the tree obtained by attaching m pendant edges to the first internal vertex through an edge and attaching n pendant edges to the second internal vertex of the path P_4.

19. Let ${\mathcal{T}_{19}}^4$ be the tree obtained by attaching m pendant edges to the first internal vertex and attaching n path of length two to the second internal vertex of the path P_4.

20. Let ${\mathcal{T}_{20}}^4$ be the tree obtained by attaching m path of length two to the first internal vertex and attaching n pendant edges to the second internal vertex of the path P_4.

21. Let ${\mathcal{T}_{21}}^5$ be the tree obtained by attaching m, l pendant edges to the first and third internal vertices and attaching l pendant edges through an edge to the second internal vertex of the path P_5.

22. Let ${\mathcal{T}_{22}}^5$ be the tree obtained by attaching n path of length two to the second internal vertex and m, l pendant edges to the first and third internal vertices of the path P_5.

Remarks 2.5:
By the above notations, we have the following remarks.
${\mathcal{T}_1}^4 \cong {\mathcal{T}_3}^4, {\mathcal{T}_4}^4 \cong {\mathcal{T}_6}^4, {\mathcal{T}_8}^4 \cong {\mathcal{T}_9}^4, {\mathcal{T}_{12}}^4 \cong {\mathcal{T}_{13}}^4$, ${\mathcal{T}_{17}}^4 \cong {\mathcal{T}_{18}}^4$ and ${\mathcal{T}_{19}}^4 \cong {\mathcal{T}_{20}}^4$.

3. MAIN RESULT

Theorem 3.1: The tree ${\mathcal{T}_1}^2$ is fuzzy quotient-3 cordial.
Proof:
Consider the tree ${\mathcal{T}_1}^2$.
$V\left({\mathcal{T}_1}^2\right) = \{x_i : 1 \leq i \leq 3\} \cup \{y_k : 1 \leq k \leq n\}$.
$E\left({\mathcal{T}_1}^2\right) = \{x_i x_{i+1} : 1 \leq i \leq 2\} \cup \{x_2 y_k : 1 \leq k \leq n\}$.
$|V| = n + 3$ and $|E| = n + 2$.
We define $\sigma : V\left({\mathcal{T}_1}^2\right) \rightarrow [0,1]$ by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$.
$\sigma(x_1) = 0.1, \sigma(x_2) = 0.3, \sigma(x_3) = 0.2$ and the values of $\sigma(y_k), 1 \leq k \leq n$ are tabulated below.

Table 3.1.1		
$k \equiv 0(mod\ 3)$	$\sigma(y_k) = 0.1$	$1 \leq k \leq n$
$k \equiv 2(mod\ 3)$	$\sigma(y_k) = 0.2$	$1 \leq k \leq n$
$k \equiv 1(mod\ 3)$	$\sigma(y_k) = 0.3$	$1 \leq k \leq n$

Table 3.1.2						
Nature of n	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$n \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}-1$
$n \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}+1$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$
$n \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert+1}{3}-1$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}-1$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}-1$

From the above table 3.1.2 it is clear that for $i, j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \neq j$ the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1.
Hence it is concluded that the tree $\mathcal{T}_1{}^2$ is fuzzy quotient-3 cordial. □

Theorem 3.2: The tree $\mathcal{T}_1{}^3$ is fuzzy quotient-3 cordial.
Proof:
Consider the tree $\mathcal{T}_1{}^3$.
$V(\mathcal{T}_1{}^3) = \{x_i : 1 \leq i \leq 4\} \cup \{y_i : 1 \leq k \leq n\}$.
$E(\mathcal{T}_1{}^3) = \{x_i x_{i+1} : 1 \leq i \leq 3\} \cup \{x_2 y_k : 1 \leq k \leq n\}$.
$|V| = n + 4$ and $|E| = n + 3$.
We define $\sigma: V(\mathcal{T}_1{}^3) \to [0,1]$ by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$
$\sigma(x_1) = 0.1, \sigma(x_2) = 0.1, \sigma(x_3) = 0.3, \sigma(x_4) = 0.2$ and the values of $\sigma(y_k), 1 \leq k \leq n$ are tabulated below.

Table 3.2.1		
$k \equiv 0(mod\ 3)$	$\sigma(y_k) = 0.1$	$1 \leq k \leq n$
$k \equiv 2(mod\ 3)$	$\sigma(y_k) = 0.2$	$1 \leq k \leq n$
$k \equiv 1(mod\ 3)$	$\sigma(y_k) = 0.3$	$1 \leq k \leq n$

Table 3.2.2						
Nature of n	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$n \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert-1}{3}+1$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$
$n \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}-1$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}-1$	$\frac{\lvert V\rvert+1}{3}-1$
$n \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}-1$

From the above table 3.2.2 it is clear that for $i, j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \neq j$ the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1.
Hence it is concluded that the tree $\mathcal{T}_1{}^3$ is fuzzy quotient-3 cordial. □

Theorem 3.3: The tree $\mathcal{T}_3{}^3$ is fuzzy quotient-3 cordial.
Proof:
Consider the tree $\mathcal{T}_3{}^3$.
$V(\mathcal{T}_3{}^3) = \{x_i : 1 \leq i \leq 4\} \cup \{y_j : 1 \leq j \leq m\} \cup \{z_i : 1 \leq k \leq n\}$.
$E(\mathcal{T}_3{}^3) = \{x_i x_{i+1} : 1 \leq i \leq 3\} \cup \{x_2 y_j : 1 \leq j \leq m\} \cup \{x_3 z_k : 1 \leq k \leq n\}$.
$|V| = m + n + 4$ and $|E| = m + n + 3$.
We define $\sigma: V(\mathcal{T}_3{}^3) \to [0,1]$ by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$.
$\sigma(x_1) = 0.1, \sigma(x_2) = 0.1, \sigma(x_3) = 0.3, \sigma(x_4) = 0.2$ and

the values of $\sigma(y_j), 1 \le j \le m$ and $\sigma(z_k), 1 \le k \le n$ are tabulated below.

Table 3.3.1			
$m \equiv 0,1,2(mod\ 3)$	$j \equiv 0(mod\ 3)$	$\sigma(y_j) = 0.1$	$1 \le j \le m$
	$j \equiv 2(mod\ 3)$	$\sigma(y_j) = 0.2$	$1 \le j \le m$
	$j \equiv 1(mod\ 3)$	$\sigma(y_j) = 0.3$	$1 \le j \le m$
$m \equiv 0,2(mod\ 3)$	$k \equiv 0(mod\ 3)$	$\sigma(z_k) = 0.1$	$1 \le k \le n$
	$k \equiv 2(mod\ 3)$	$\sigma(z_k) = 0.2$	$1 \le k \le n$
	$k \equiv 1(mod\ 3)$	$\sigma(z_k) = 0.3$	$1 \le k \le n$
$m \equiv 1(mod\ 3)$	$k \equiv 0(mod\ 3)$	$\sigma(z_k) = 0.1$	$1 \le k \le n$
	$k \equiv 1(mod\ 3)$	$\sigma(z_k) = 0.2$	$1 \le k \le n$
	$k \equiv 2(mod\ 3)$	$\sigma(z_k) = 0.3$	$1 \le k \le n$

Table 3.3.2						
Nature of m and n	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$m \equiv 0(mod\ 3)$ $n \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 0(mod\ 3)$ $n \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$
$m \equiv 0(mod\ 3)$ $n \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$
$m \equiv 1(mod\ 3)$ $n \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3} - 1$
$m \equiv 1(mod\ 3)$ $n \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$
$m \equiv 1(mod\ 3)$ $n \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 2(mod\ 3)$ $n \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$
$m \equiv 2(mod\ 3)$ $n \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 2(mod\ 3)$ $n \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$

From the above table 3.3.2 it is clear that for $i, j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \ne j$ the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1.
Hence it is concluded that the tree $\mathcal{T}_3{}^3$ is fuzzy quotient-3 cordial. □

Theorem 3.4: The tree $\mathcal{T}_4{}^3$ is fuzzy quotient-3 cordial.
Proof:
Consider the tree $\mathcal{T}_4{}^3$.
$V(\mathcal{T}_4{}^3) = \{x_i : 1 \le i \le 3\} \cup \{x\} \cup \{y_k : 1 \le k \le n\}$.
$E(\mathcal{T}_4{}^3) = \{x_i x_{i+1} : 1 \le i \le 2\} \cup \{x_2 x\} \cup \{xy_k : 1 \le k \le n\}$.
$|V| = n + 4$ and $|E| = n + 3$.
We define $\sigma : V(\mathcal{T}_5{}^3) \to [0,1]$ by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$.
$\sigma(x) = 0.1, \sigma(x_1) = 0.3, \sigma(x_2) = 0.1, \sigma(x_3) = 0.2$ and the values of $\sigma(y_k), 1 \le k \le n$ are tabulated below.

Table 3.4.1		
$k \equiv 0(mod\ 3)$	$\sigma(y_k) = 0.1$	$1 \le k \le n$
$k \equiv 2(mod\ 3)$	$\sigma(y_k) = 0.2$	$1 \le k \le n$
$k \equiv 1(mod\ 3)$	$\sigma(y_k) = 0.3$	$1 \le k \le n$

Table 3.4.2						
Nature of n	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$n \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$n \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3} - 1$
$n \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$

From the above table 3.4.2 it is clear that for $i, j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \neq j$ the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1.
Hence it is concluded that the tree ${\mathcal{T}_4}^3$ is fuzzy quotient-3 cordial. □

Theorem 3.5: The tree ${\mathcal{T}_1}^4$ is fuzzy quotient-3 cordial.
Proof:
Consider the tree ${\mathcal{T}_1}^4$.
$V\left({\mathcal{T}_1}^4\right) = \{x_i : 1 \le i \le 5\} \cup \{y_k : 1 \le k \le n\}$.
$E\left({\mathcal{T}_1}^4\right) = \{x_i x_{i+1} : 1 \le i \le 4\} \cup \{x_2 y_k : 1 \le k \le n\}$.
$|V| = n + 5$ and $|E| = n + 4$.
We define $\sigma : V\left({\mathcal{T}_1}^4\right) \to [0,1]$ by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$.
$\sigma(x_1) = 0.2, \sigma(x_2) = 0.3, \sigma(x_3) = 0.1, \sigma(x_4) = 0.1, \sigma(x_5) = 0.3$ and the values of $\sigma(y_k), 1 \le k \le n$ are tabulated below.

Table 3.5.1		
$k \equiv 0(mod\ 3)$	$\sigma(y_k) = 0.1$	$1 \le k \le n$
$k \equiv 1(mod\ 3)$	$\sigma(y_k) = 0.2$	$1 \le k \le n$
$k \equiv 2(mod\ 3)$	$\sigma(y_k) = 0.3$	$1 \le k \le n$

Table 3.5.2						
Nature of n	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$n \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3} - 1$
$n \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$
$n \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$

From the above table 3.5.2 it is clear that for $i, j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \neq j$ the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1.
Hence it is concluded that the tree ${\mathcal{T}_1}^4$ is fuzzy quotient-3 cordial. □

Theorem 3.6: The tree $\mathcal{T}_2{}^4$ is fuzzy quotient-3 cordial.
Proof:
Consider the tree $\mathcal{T}_2{}^4$.
$V(\mathcal{T}_2{}^4) = \{x_i : 1 \leq i \leq 5\} \cup \{y_k : 1 \leq k \leq n\}$.
$E(\mathcal{T}_2{}^4) = \{x_i x_{i+1} : 1 \leq i \leq 4\} \cup \{x_3 y_k : 1 \leq k \leq n\}$.
$|V| = n + 5$ and $|E| = n + 4$.
We define $\sigma: V(\mathcal{T}_2{}^4) \to [0,1]$ by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$. $\sigma(x_1) = 0.2, \sigma(x_2) = 0.3$, $\sigma(x_3) = 0.1, \sigma(x_4) = 0.1, \sigma(x_5) = 0.3$ and the values of $\sigma(y_k), 1 \leq k \leq n$ are tabulated below.

Table 3.6.1		
$k \equiv 2(mod\ 3)$	$\sigma(y_k) = 0.1$	$1 \leq k \leq n$
$k \equiv 1(mod\ 3)$	$\sigma(y_k) = 0.2$	$1 \leq k \leq n$
$k \equiv 0(mod\ 3)$	$\sigma(y_k) = 0.3$	$1 \leq k \leq n$

Table 3.6.2						
Nature of n	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$n \equiv 0(mod\ 3)$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3} - 1$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3} - 1$	$\frac{\|V\|+1}{3} - 1$
$n \equiv 1(mod\ 3)$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3} - 1$
$n \equiv 2(mod\ 3)$	$\frac{\|V\|-1}{3} + 1$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$

From the above table 3.6.2 it is clear that for $i, j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \neq j$ the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1.
Hence it is concluded that the tree $\mathcal{T}_2{}^4$ is fuzzy quotient-3 cordial. □

Theorem 3.7: The tree $\mathcal{T}_4{}^4$ is fuzzy quotient-3 cordial.
Proof:
Consider the tree $\mathcal{T}_4{}^4$.
$V(\mathcal{T}_4{}^4) = \{x_i : 1 \leq i \leq 5\} \cup \{y_j : 1 \leq j \leq m\} \cup \{z_i : 1 \leq k \leq n\}$.
$E(\mathcal{T}_4{}^4) = \{x_i x_{i+1} : 1 \leq i \leq 4\} \cup \{x_2 y_j : 1 \leq j \leq m\} \cup \{x_3 z_k : 1 \leq k \leq n\}$.
$|V| = m + n + 5$ and $|E| = m + n + 4$.
We define $\sigma: V(\mathcal{T}_4{}^4) \to [0,1]$ by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$.
$\sigma(x_1) = 0.2, \sigma(x_2) = 0.3, \sigma(x_3) = 0.1, \sigma(x_4) = 0.1, \sigma(x_5) = 0.3$ and the values of $\sigma(y_j), 1 \leq j \leq m$ and $\sigma(z_k), 1 \leq k \leq n$ are tabulated below.

Table 3.7.1			
$j \equiv 0(mod\ 3)$		$\sigma(y_j) = 0.1$	$1 \leq j \leq m$
$j \equiv 1(mod\ 3)$		$\sigma(y_j) = 0.2$	$1 \leq j \leq m$
$j \equiv 2(mod\ 3)$		$\sigma(y_j) = 0.3$	$1 \leq j \leq m$
$m \equiv 0(mod\ 3)$	$k \equiv 2(mod\ 3)$	$\sigma(z_k) = 0.1$	$1 \leq k \leq n$
	$k \equiv 1(mod\ 3)$	$\sigma(z_k) = 0.2$	$1 \leq k \leq n$
	$k \equiv 0(mod\ 3)$	$\sigma(z_k) = 0.3$	$1 \leq k \leq n$
$m \equiv 1,2(mod\ 3)$	$k \equiv 1(mod\ 3)$	$\sigma(z_k) = 0.1$	$1 \leq k \leq n$
	$k \equiv 2(mod\ 3)$	$\sigma(z_k) = 0.2$	$1 \leq k \leq n$
	$k \equiv 0(mod\ 3)$	$\sigma(z_k) = 0.3$	$1 \leq k \leq n$

Table 3.7.2						
Nature of m and n	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$m \equiv 0(mod\ 3)$ $n \equiv 0(mod\ 3)$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}-1$
$m \equiv 0(mod\ 3)$ $n \equiv 1(mod\ 3)$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}-1$
$m \equiv 0(mod\ 3)$ $n \equiv 2(mod\ 3)$	$\frac{\|V\|-1}{3}+1$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$
$m \equiv 1(mod\ 3)$ $n \equiv 0(mod\ 3)$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}-1$
$m \equiv 1(mod\ 3)$ $n \equiv 1(mod\ 3)$	$\frac{\|V\|-1}{3}+1$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$
$m \equiv 1(mod\ 3)$ $n \equiv 2(mod\ 3)$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$
$m \equiv 2(mod\ 3)$ $n \equiv 0(mod\ 3)$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}+1$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$
$m \equiv 2(mod\ 3)$ $n \equiv 1(mod\ 3)$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}$
$m \equiv 2(mod\ 3)$ $n \equiv 2(mod\ 3)$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}-1$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$

From the above table 3.7.2 it is clear that for $i, j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \neq j$ the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1.
Hence it is concluded that the tree ${\mathcal{T}_4}^4$ is fuzzy quotient-3 cordial. □

Theorem 3.8: The tree ${\mathcal{T}_5}^4$ is fuzzy quotient-3 cordial.
Proof:
Consider the tree ${\mathcal{T}_5}^4$.
$V\left({\mathcal{T}_5}^4\right) = \{x_i : 1 \leq i \leq 5\} \cup \{y_j : 1 \leq j \leq m\} \cup \{z_i : 1 \leq k \leq n\}$.
$E\left({\mathcal{T}_5}^4\right) = \{x_i x_{i+1} : 1 \leq i \leq 4\} \cup \{x_2 y_j : 1 \leq j \leq m\} \cup \{x_4 z_k : 1 \leq k \leq n\}$.
$|V| = m + n + 5$ and $|E| = m + n + 4$.
The proof follows from the theorem 3.7 □

Theorem 3.9: The tree ${\mathcal{T}_7}^4$ is fuzzy quotient-3 cordial.
Proof:
Consider the tree ${\mathcal{T}_7}^4$.
$V\left({\mathcal{T}_7}^4\right) = \{v_h : 1 \leq h \leq 5\} \cup \{x_i : 1 \leq i \leq m\} \cup \{y_j : 1 \leq j \leq n\} \cup \{z_i : 1 \leq k \leq l\}$.
$E\left({\mathcal{T}_7}^4\right) = \{v_h v_{h+1} : 1 \leq h \leq 4\} \cup \{v_2 x_i : 1 \leq i \leq m\} \cup \{v_3 y_j : 1 \leq j \leq n\}$
$\cup \{v_4 z_k : 1 \leq k \leq l\}$.
$|V| = m + n + l + 5$ and $|E| = m + n + l + 4$.
We define $\sigma : V\left({\mathcal{T}_7}^4\right) \to [0,1]$ by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$.
$\sigma(v_1) = 0.2, \sigma(v_2) = 0.3, \sigma(v_3) = 0.1, \sigma(v_4) = 0.1, \sigma(v_5) = 0.3$ and
the values of $\sigma(y_j), 1 \leq j \leq n$ and $\sigma(z_k), 1 \leq k \leq l$ are tabulated below.

Table 3.9.1			
$i \equiv 0(mod\ 3)$	$\sigma(x_i) = 0.1$		$1 \leq i \leq m$
$i \equiv 1(mod\ 3)$	$\sigma(x_i) = 0.2$		$1 \leq i \leq m$
$i \equiv 2(mod\ 3)$	$\sigma(x_i) = 0.3$		$1 \leq i \leq m$
$m \equiv 0,2(mod\ 3)$	$j \equiv 2(mod\ 3)$	$\sigma(y_j) = 0.1$	$1 \leq j \leq n$
	$j \equiv 1(mod\ 3)$	$\sigma(y_j) = 0.2$	$1 \leq j \leq n$
	$j \equiv 0(mod\ 3)$	$\sigma(y_j) = 0.3$	$1 \leq j \leq n$
$m \equiv 1(mod\ 3)$	$j \equiv 1(mod\ 3)$	$\sigma(y_j) = 0.1$	$1 \leq j \leq n$
	$j \equiv 2(mod\ 3)$	$\sigma(y_j) = 0.2$	$1 \leq j \leq n$
	$j \equiv 0(mod\ 3)$	$\sigma(y_j) = 0.3$	$1 \leq j \leq n$
$m \equiv 0(mod\ 3)$ $n \equiv 0(mod\ 3)$	$k \equiv 2(mod\ 3)$	$\sigma(z_k) = 0.1$	$1 \leq k \leq l$
	$k \equiv 1(mod\ 3)$	$\sigma(z_k) = 0.2$	$1 \leq k \leq l$
	$k \equiv 0(mod\ 3)$	$\sigma(z_k) = 0.3$	$1 \leq k \leq l$
$m \equiv 0(mod\ 3)$ $n \equiv 1(mod\ 3)$ and $m \equiv 1(mod\ 3)$ $n \equiv 0(mod\ 3)$	$k \equiv 1(mod\ 3)$	$\sigma(z_k) = 0.1$	$1 \leq k \leq l$
	$k \equiv 2(mod\ 3)$	$\sigma(z_k) = 0.2$	$1 \leq k \leq l$
	$k \equiv 0(mod\ 3)$	$\sigma(z_k) = 0.3$	$1 \leq k \leq l$
$m \equiv 0(mod\ 3)$ $n \equiv 2(mod\ 3)$ and $m \equiv 1(mod\ 3)$ $n \equiv 1(mod\ 3)$	$k \equiv 0(mod\ 3)$	$\sigma(z_k) = 0.1$	$1 \leq k \leq l$
	$k \equiv 1(mod\ 3)$	$\sigma(z_k) = 0.2$	$1 \leq k \leq l$
	$k \equiv 2(mod\ 3)$	$\sigma(z_k) = 0.3$	$1 \leq k \leq l$
$m \equiv 1(mod\ 3)$ $n \equiv 2(mod\ 3)$ and $m \equiv 2(mod\ 3)$ $n \equiv 0,2(mod\ 3)$	$k \equiv 2(mod\ 3)$	$\sigma(z_k) = 0.1$	$1 \leq k \leq l$
	$k \equiv 0(mod\ 3)$	$\sigma(z_k) = 0.2$	$1 \leq k \leq l$
	$k \equiv 1(mod\ 3)$	$\sigma(z_k) = 0.3$	$1 \leq k \leq l$
$m \equiv 2(mod\ 3)$ $n \equiv 1(mod\ 3)$	$k \equiv 1(mod\ 3)$	$\sigma(z_k) = 0.1$	$1 \leq k \leq l$
	$k \equiv 0(mod\ 3)$	$\sigma(z_k) = 0.2$	$1 \leq k \leq l$
	$k \equiv 2(mod\ 3)$	$\sigma(z_k) = 0.3$	$1 \leq k \leq l$

Table 3.9.2						
Nature of m, n *and* l	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$m \equiv 0(mod\ 3), n \equiv 0(mod\ 3),$ $l \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3} - 1$
$m \equiv 0(mod\ 3), n \equiv 0(mod\ 3),$ $l \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$
$m \equiv 0(mod\ 3), n \equiv 0(mod\ 3),$ $l \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 0(mod\ 3), n \equiv 1(mod\ 3),$ $l \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$
$m \equiv 0(mod\ 3), n \equiv 1(mod\ 3),$ $l \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 0(mod\ 3), n \equiv 1(mod\ 3),$ $l \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$

$m \equiv 0(mod\ 3), n \equiv 2(mod\ 3), l \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 0(mod\ 3), n \equiv 2(mod\ 3), l \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$
$m \equiv 0(mod\ 3), n \equiv 2(mod\ 3), l \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$
$m \equiv 1(mod\ 3), n \equiv 0(mod\ 3), l \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$
$m \equiv 1(mod\ 3), n \equiv 0(mod\ 3), l \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 1(mod\ 3), n \equiv 0(mod\ 3), l \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3} - 1$
$m \equiv 1(mod\ 3), n \equiv 1(mod\ 3), l \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 1(mod\ 3), n \equiv 1(mod\ 3), l \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$
$m \equiv 1(mod\ 3), n \equiv 1(mod\ 3), l \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$
$m \equiv 1(mod\ 3), n \equiv 2(mod\ 3), l \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$
$m \equiv 1(mod\ 3), n \equiv 2(mod\ 3), l \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$
$m \equiv 1(mod\ 3), n \equiv 2(mod\ 3), l \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 2(mod\ 3), n \equiv 0(mod\ 3), l \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 2(mod\ 3), n \equiv 0(mod\ 3), l \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$
$m \equiv 2(mod\ 3), n \equiv 0(mod\ 3), l \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$
$m \equiv 2(mod\ 3), n \equiv 1(mod\ 3), l \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$
$m \equiv 2(mod\ 3), n \equiv 1(mod\ 3), l \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$
$m \equiv 2(mod\ 3), n \equiv 1(mod\ 3), l \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 2(mod\ 3), n \equiv 2(mod\ 3), l \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$

$m \equiv 2(mod\ 3), n \equiv 2(mod\ 3),$ $l \equiv 1(mod\ 3)$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}+1$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$
$m \equiv 2(mod\ 3), n \equiv 2(mod\ 3),$ $l \equiv 2(mod\ 3)$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$

From the above table 3.9.2 it is clear that for $i, j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \neq j$ the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1.
Hence it is concluded that the tree ${\mathcal{T}_7}^4$ is fuzzy quotient-3 cordial. □

Theorem 3.10: The tree ${\mathcal{T}_8}^4$ is fuzzy quotient-3 cordial.
Proof:
Consider the tree ${\mathcal{T}_8}^4$.
$V\left({\mathcal{T}_8}^4\right) = \{x_i : 1 \leq i \leq 4\} \cup \{x\} \cup \{y_k : 1 \leq k \leq n\}$.
$E\left({\mathcal{T}_8}^4\right) = \{x_i x_{i+1} : 1 \leq i \leq 3\} \cup \{x_2 x\} \cup \{x y_k : 1 \leq k \leq n\}$.
$|V| = n + 5$ and $|E| = n + 4$.
We define $\sigma: V\left({\mathcal{T}_{12}}^4\right) \rightarrow [0,1]$ by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$.
$\sigma(x) = 0.3, \sigma(x_1) = 0.2, \sigma(x_2) = 0.1, \sigma(x_3) = 0.1, \sigma(x_4) = 0.3$ and the values of $\sigma(y_k), 1 \leq k \leq n$ are tabulated below.

Table 3.10.1		
$k \equiv 0(mod\ 3)$	$\sigma(y_k) = 0.1$	$1 \leq k \leq n$
$k \equiv 2(mod\ 3)$	$\sigma(y_k) = 0.2$	$1 \leq k \leq n$
$k \equiv 1(mod\ 3)$	$\sigma(y_k) = 0.3$	$1 \leq k \leq n$

Table 3.10.2						
Nature of n	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$n \equiv 0(mod\ 3)$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}-1$
$n \equiv 1(mod\ 3)$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}-1$
$n \equiv 2(mod\ 3)$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}+1$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$

From the above table 3.10.2 it is clear that for $i, j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \neq j$ the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1.
Hence it is concluded that the tree ${\mathcal{T}_8}^4$ is fuzzy quotient-3 cordial. □

Theorem 3.11: The tree ${\mathcal{T}_{10}}^4$ is fuzzy quotient-3 cordial.
Proof:
Consider the tree ${\mathcal{T}_{10}}^4$.
$V\left({\mathcal{T}_{10}}^4\right) = \{x_i : 1 \leq i \leq 5\} \cup \{x\} \cup \{y_k : 1 \leq k \leq n\}$.
$E\left({\mathcal{T}_{10}}^4\right) = \{x_i x_{i+1} : 1 \leq i \leq 4\} \cup \{x_2 x\} \cup \{x y_k : 1 \leq k \leq n\}$.
$|V| = n + 6$ and $|E| = n + 5$.
We define $\sigma: V\left({\mathcal{T}_{15}}^4\right) \rightarrow [0,1]$ by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$.
$\sigma(x) = 0.3, \sigma(x_1) = 0.2, \sigma(x_2) = 0.1, \sigma(x_3) = 0.3, \sigma(x_4) = 0.1, \sigma(x_5) = 0.2$ and the values of $\sigma(y_k), 1 \leq k \leq n$ are tabulated below.

Table 3.11.1		
$k \equiv 0(mod\ 3)$	$\sigma(y_k) = 0.1$	$1 \le k \le n$
$k \equiv 2(mod\ 3)$	$\sigma(y_k) = 0.2$	$1 \le k \le n$
$k \equiv 1(mod\ 3)$	$\sigma(y_k) = 0.3$	$1 \le k \le n$

Table 3.11.2						
Nature of n	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$n \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}-1$
$n \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}+1$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$
$n \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert+1}{3}-1$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}-1$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}-1$

From the above table 3.11.2 it is clear that for $i,j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \ne j$ the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1.
Hence it is concluded that the tree ${\mathcal{T}_{10}}^4$ is fuzzy quotient-3 cordial. □

Theorem 3.12: The tree ${\mathcal{T}_{11}}^4$ is fuzzy quotient-3 cordial.
Proof:
Consider the tree ${\mathcal{T}_{11}}^4$.
$V\left({\mathcal{T}_{11}}^4\right) = \{x_i : 1 \le i \le 3\} \cup \{{y_j}^k : 1 \le j \le 2, 1 \le k \le n\}$
$E\left({\mathcal{T}_{11}}^4\right) = \{x_i x_{i+1} : 1 \le i \le 2\} \cup \{x_2 {y_j}^k : 1 \le j \le 2, 1 \le k \le n\}$
$|V| = 2n + 3$ and $|E| = 2n + 2$.
We define $\sigma: V\left({\mathcal{T}_4}^3\right) \to [0,1]$ by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$.
$\sigma(x_1) = 0.3, \sigma(x_2) = 0.1, \sigma(x_3) = 0.2$ and the values of $\sigma\left({y_j}^k\right), 1 \le j \le 2, 1 \le k \le n$ are tabulated below.

Table 3.12.1			
$n \equiv 0,2(mod\ 3)$	$k \equiv 2(mod\ 3)$	$\sigma({y_1}^k) = 0.1$	$1 \le k \le n$
	$k \equiv 0,1(mod\ 3)$	$\sigma({y_1}^k) = 0.3$	$1 \le k \le n$
	$k \equiv 2(mod\ 3)$	$\sigma({y_2}^k) = 0.1$	$1 \le k \le n$
	$k \equiv 0,1(mod\ 3)$	$\sigma({y_2}^k) = 0.2$	$1 \le k \le n$
$n \equiv 1(mod\ 3)$	$k \equiv 0,1(mod\ 3)$	$\sigma({y_1}^k) = 0.1$	$1 \le k \le n$
	$k \equiv 2(mod\ 3)$	$\sigma({y_1}^k) = 0.3$	$1 \le k \le n$
	$k \equiv 1,2(mod\ 3)$	$\sigma({y_2}^k) = 0.2$	$1 \le k \le n$
	$k \equiv 0(mod\ 3)$	$\sigma({y_2}^k) = 0.3$	$1 \le k \le n$

Table 3.12.2						
Nature of n	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$n \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}-1$
$n \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}-1$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}-1$	$\frac{\lvert V\rvert+1}{3}-1$
$n \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert-1}{3}+1$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$

From the above table 3.12.2 it is clear that for $i,j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \ne j$ the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1.
Hence it is concluded that the tree ${\mathcal{T}_{11}}^4$ is fuzzy quotient-3 cordial. □

Theorem 3.13: The tree ${\mathcal{T}_{12}}^4$ is fuzzy quotient-3 cordial.
Proof:
Consider the tree ${\mathcal{T}_{12}}^4$.
$V\left({\mathcal{T}_{12}}^4\right) = \{x_i : 1 \leq i \leq 4\} \cup \{{y_j}^k : 1 \leq j \leq n, 1 \leq k \leq 2\}$.
$E\left({\mathcal{T}_{12}}^4\right) = \{x_i x_{i+1} : 1 \leq i \leq 3\} \cup \{x_2 {y_j}^1 : 1 \leq j \leq n\} \cup \{{y_j}^1 {y_j}^{1+k} : 1 \leq j \leq n, k = 1\}$.
$|V| = 2n + 4$ and $|E| = 2n + 3$.
We define $\sigma: V\left({\mathcal{T}_9}^4\right) \to [0,1]$ by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$.
$\sigma(x_1) = 0.2, \sigma(x_2) = 0.1, \sigma(x_3) = 0.1, \sigma(x_4) = 0.3$
and the values of $\sigma\left({y_j}^k\right), 1 \leq j \leq n, 1 \leq k \leq 2$ are tabulated below.

Table 3.13.1		
$j \equiv 0,2 (mod\ 3)$	$\sigma\left({y_j}^1\right) = 0.1$	$1 \leq j \leq n$
$j \equiv 1 (mod\ 3)$	$\sigma\left({y_j}^1\right) = 0.3$	$1 \leq j \leq n$
$j \equiv 0 (mod\ 3)$	$\sigma\left({y_j}^2\right) = 0.3$	$1 \leq j \leq n$
$j \equiv 1,2 (mod\ 3)$	$\sigma\left({y_j}^2\right) = 0.2$	$1 \leq j \leq n$

Table 3.13.2						
Nature of n	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$n \equiv 0 (mod\ 3)$	$\frac{\|V\|-1}{3}+1$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$
$n \equiv 1 (mod\ 3)$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}-1$
$n \equiv 2 (mod\ 3)$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$

From the above table 3.13.2 it is clear that for $i, j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \neq j$ the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1.
Hence it is concluded that the tree ${\mathcal{T}_{12}}^4$ is fuzzy quotient-3 cordial. □

Theorem 3.14: The tree ${\mathcal{T}_{14}}^4$ is fuzzy quotient-3 cordial.
Proof:
Consider the tree ${\mathcal{T}_{14}}^4$.
$V\left({\mathcal{T}_{14}}^4\right) = \{x_i : 1 \leq i \leq 5\} \cup \left\{{y_j}^k : 1 \leq j \leq n, 1 \leq k \leq 2\right\}$.
$E\left({\mathcal{T}_{14}}^4\right) = \{x_i x_{i+1} : 1 \leq i \leq 4\} \cup \{x_3 {y_j}^1 : 1 \leq j \leq n\} \cup \{{y_j}^k {y_j}^{k+1} : 1 \leq j \leq n, k = 1\}$.
$|V| = 2n + 5$ and $|E| = 2n + 4$.
We define $\sigma: V\left({\mathcal{T}_{18}}^4\right) \to [0,1]$ by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$.
$\sigma(x_1) = 0.2, \sigma(x_2) = 0.1, \sigma(x_3) = 0.1, \sigma(x_4) = 0.3, \sigma(x_5) = 0.2$ and
the values of $\sigma\left({y_j}^k\right), 1 \leq j \leq n, 1 \leq k \leq 2$ are tabulated below.

Table 3.14.1		
$j \equiv 2 (mod\ 3)$	$\sigma\left({y_j}^1\right) = 0.3$	$1 \leq j \leq n$
$j \equiv 0,1 (mod\ 3)$	$\sigma\left({y_j}^1\right) = 0.1$	$1 \leq j \leq n$
$j \equiv 0,2 (mod\ 3)$	$\sigma\left({y_j}^2\right) = 0.2$	$1 \leq j \leq n$
$j \equiv 1 (mod\ 3)$	$\sigma\left({y_j}^2\right) = 0.3$	$1 \leq j \leq n$

Table 3.14.2						
Nature of n	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$n \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}-1$	$\frac{\lvert V\rvert+1}{3}-1$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}-1$
$n \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert-1}{3}+1$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$
$n \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}-1$

From the above table 3.14.2 it is clear that for $i,j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \neq j$ the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1.
Hence it is concluded that the tree ${\mathcal{T}_{14}}^4$ is fuzzy quotient-3 cordial. □

Theorem 3.15: The tree ${\mathcal{T}_{15}}^4$ is fuzzy quotient-3 cordial.
Proof:
Consider the tree ${\mathcal{T}_{15}}^4$.
$V\left({\mathcal{T}_{15}}^4\right) = \{x_i : 1 \leq i \leq 3\} \cup \{y\} \cup \{{y_j}^k : 1 \leq j \leq n, 1 \leq k \leq 2\}$.
$E\left({\mathcal{T}_{15}}^4\right) = \{x_i x_{i+1} : 1 \leq i \leq 2\} \cup \{x_2 y\} \cup \{y{y_j}^1 : 1 \leq j \leq 2\} \cup \{{y_j}^1 {y_j}^{1+k} : 1 \leq j \leq n, k = 1\}$.
$|V| = 2n + 4$ and $|E| = 2n + 3$.
We define $\sigma: V\left({\mathcal{T}_{16}}^4\right) \to [0,1]$ by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$.
$\sigma(y) = 0.1, \sigma(x_1) = 0.3, \sigma(x_2) = 0.1, \sigma(x_3) = 0.2$ and
the values of $\sigma\left({y_j}^k\right), 1 \leq j \leq n, 1 \leq k \leq 2$ are tabulated below.

Table 3.15.1			
$n \equiv 0,1(mod\ 3)$	$j \equiv 2(mod\ 3)$	$\sigma\left({y_j}^1\right) = 0.1$	$1 \leq j \leq n$
	$j \equiv 0,1(mod\ 3)$	$\sigma\left({y_j}^1\right) = 0.3$	$1 \leq j \leq n$
	$j \equiv 0,1(mod\ 3)$	$\sigma\left({y_j}^2\right) = 0.2$	$1 \leq j \leq n$
	$j \equiv 2(mod\ 3)$	$\sigma\left({y_j}^2\right) = 0.1$	$1 \leq j \leq n$
$n \equiv 2(mod\ 3)$	$j \equiv 0,1(mod\ 3)$	$\sigma\left({y_j}^1\right) = 0.1$	$1 \leq j \leq n$
	$j \equiv 0,2(mod\ 3)$	$\sigma\left({y_j}^1\right) = 0.3$	$1 \leq j \leq n$
	$j \equiv 1,2(mod\ 3)$	$\sigma\left({y_j}^2\right) = 0.2$	$1 \leq j \leq n$
	$j \equiv 0(mod\ 3)$	$\sigma\left({y_j}^2\right) = 0.3$	$1 \leq j \leq n$

Table 3.15.2						
Nature of n	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$n \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert-1}{3}+1$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$
$n \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}-1$
$n \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}-1$	$\frac{\lvert V\rvert+1}{3}-1$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}-1$

From the above table 3.15.2 it is clear that for $i,j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \neq j$ the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1.
Hence it is concluded that the tree ${\mathcal{T}_{15}}^4$ is fuzzy quotient-3 cordial. □

Theorem 3.16: The tree ${\mathcal{T}_{16}}^4$ is fuzzy quotient-3 cordial.
Proof:
Consider the tree ${\mathcal{T}_{16}}^4$.
$V\left({\mathcal{T}_{16}}^4\right) = \{x_i : 1 \leq i \leq 3\} \cup \{x, y\} \cup \{y_k : 1 \leq k \leq n\}$.
$E\left({\mathcal{T}_{16}}^4\right) = \{x_i x_{i+1} : 1 \leq i \leq 2\} \cup \{x_2 x, xy\} \cup \{yy_k : 1 \leq k \leq n\}$.
$|V| = n + 5$ and $|E| = n + 4$.
We define $\sigma : V\left({\mathcal{T}_{17}}^4\right) \rightarrow [0,1]$ by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$.
$\sigma(x) = 0.1, \sigma(y) = 0.3, \sigma(x_1) = 0.3, \sigma(x_2) = 0.1, \sigma(x_3) = 0.2$ and the values of $\sigma(y_k), 1 \leq k \leq n$ are tabulated below.

Table 3.16.1		
$k \equiv 0(mod\ 3)$	$\sigma(y_k) = 0.1$	$1 \leq k \leq n$
$k \equiv 1(mod\ 3)$	$\sigma(y_k) = 0.2$	$1 \leq k \leq n$
$k \equiv 2(mod\ 3)$	$\sigma(y_k) = 0.3$	$1 \leq k \leq n$

Table 3.16.2						
Nature of n	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$n \equiv 0(mod\ 3)$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3} - 1$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3} - 1$	$\frac{\|V\|+1}{3} - 1$
$n \equiv 1(mod\ 3)$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3} - 1$
$n \equiv 2(mod\ 3)$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3} + 1$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$

From the above table 3.16.2 it is clear that for $i, j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \neq j$ the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1.
Hence it is concluded that the tree ${\mathcal{T}_{16}}^4$ is fuzzy quotient-3 cordial. □

Theorem 3.17: The tree ${\mathcal{T}_{17}}^4$ is fuzzy quotient-3 cordial.
Proof:
Consider the tree ${\mathcal{T}_{17}}^4$.
$V\left({\mathcal{T}_{17}}^4\right) = \{v_h : 1 \leq h \leq 4\} \cup \{x_i : 1 \leq i \leq m\} \cup \{y_j : 1 \leq j \leq n\} \cup \{y\}$.
$E\left({\mathcal{T}_{17}}^4\right) = \{v_h v_{h+1} : 1 \leq h \leq 3\} \cup \{v_2 x_i : 1 \leq i \leq m\} \cup \{v_3 y\} \cup \{yy_j : 1 \leq j \leq n\}$.
$|V| = m + n + 5$ and $|E| = m + n + 4$.
We define $\sigma : V\left({\mathcal{T}_{19}}^4\right) \rightarrow [0,1]$ by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$.
$\sigma(y) = 0.1, \sigma(v_1) = 0.2, \sigma(v_2) = 0.1, \sigma(v_3) = 0.3, \sigma(v_4) = 0.3$ and the values of $\sigma(x_i), 1 \leq i \leq m, \sigma(y_j), 1 \leq j \leq n$ are tabulated below.

Table 3.17.1			
$i \equiv 1(mod\ 3)$		$\sigma(x_i) = 0.2$	$1 \leq i \leq m$
$i \equiv 2(mod\ 3)$		$\sigma(x_i) = 0.1$	$1 \leq i \leq m$
$i \equiv 0(mod\ 3)$		$\sigma(x_i) = 0.3$	$1 \leq i \leq m$
$m \equiv 0(mod\ 3)$	$j \equiv 2(mod\ 3)$	$\sigma(y_j) = 0.1$	$1 \leq j \leq n$
	$j \equiv 1(mod\ 3)$	$\sigma(y_j) = 0.2$	$1 \leq j \leq n$
	$j \equiv 0(mod\ 3)$	$\sigma(y_j) = 0.3$	$1 \leq j \leq n$
$m \equiv 1(mod\ 3)$	$j \equiv 1(mod\ 3)$	$\sigma(y_j) = 0.1$	$1 \leq j \leq n$
	$j \equiv 2(mod\ 3)$	$\sigma(y_j) = 0.2$	$1 \leq j \leq n$
	$j \equiv 0(mod\ 3)$	$\sigma(y_j) = 0.3$	$1 \leq j \leq n$
$m \equiv 2(mod\ 3)$	$j \equiv 0(mod\ 3)$	$\sigma(y_j) = 0.1$	$1 \leq j \leq n$
	$j \equiv 1(mod\ 3)$	$\sigma(y_j) = 0.2$	$1 \leq j \leq n$
	$j \equiv 2(mod\ 3)$	$\sigma(y_j) = 0.3$	$1 \leq j \leq n$

Table 3.17.2						
Nature of m and n	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$m \equiv 0(mod\ 3)$ $n \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}-1$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}-1$	$\frac{\lvert V\rvert+1}{3}-1$
$m \equiv 0(mod\ 3)$ $n \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}-1$
$m \equiv 0(mod\ 3)$ $n \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert-1}{3}+1$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$
$m \equiv 1(mod\ 3)$ $n \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert-1}{3}+1$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$
$m \equiv 1(mod\ 3)$ $n \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}-1$	$\frac{\lvert V\rvert+1}{3}-1$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}-1$
$m \equiv 1(mod\ 3)$ $n \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}-1$
$m \equiv 2(mod\ 3)$ $n \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert+1}{3}$	$\frac{\lvert V\rvert+1}{3}-1$	$\frac{\lvert V\rvert+1}{3}-1$	$\frac{\lvert V\rvert+1}{3}-1$	$\frac{\lvert V\rvert+1}{3}-1$	$\frac{\lvert V\rvert+1}{3}-1$
$m \equiv 2(mod\ 3)$ $n \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}-1$
$m \equiv 2(mod\ 3)$ $n \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert-1}{3}+1$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$	$\frac{\lvert V\rvert-1}{3}$

From the above table 3.17.2 it is clear that for $i,j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \neq j$ the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1.
Hence it is concluded that the tree ${\mathcal{T}_{17}}^4$ is fuzzy quotient-3 cordial. □

Theorem 3.18: The tree ${\mathcal{T}_{19}}^4$ is fuzzy quotient-3 cordial.
Proof:
Consider the tree ${\mathcal{T}_{19}}^4$.
$V\left({\mathcal{T}_{19}}^4\right) = \{v_h : 1 \le h \le 4\} \cup \{x_i : 1 \le i \le m\} \cup \{y\} \cup \left\{{y_j}^1 : 1 \le j \le n\right\} \cup \{{y_j}^2, 1 \le j \le n\}$.
$E\left({\mathcal{T}_{19}}^4\right) = \{v_h v_{h+1} : 1 \le h \le 3\} \cup \{v_2 x_i : 1 \le i \le m\} \cup \left\{v_3 {y_j}^1 : 1 \le j \le n\right\} \cup \{{y_j}^1 {y_j}^2, 1 \le j \le n\}$.
$|V| = m + 2n + 4$ and $|E| = m + 2n + 3$.
We define $\sigma: V\left({\mathcal{T}_{21}}^4\right) \to [0,1]$ by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$.
$\sigma(v_1) = 0.2$, $\sigma(v_2) = 0.1$, $\sigma(v_3) = 0.1$, $\sigma(v_4) = 0.3$ and the values of $\sigma(x_i), 1 \le i \le m, \sigma\left({y_j}^k\right), 1 \le j \le n, k = 1,2$ are tabulated below.

Table 3.18.1			
$i \equiv 1(mod\ 3)$		$\sigma(x_i) = 0.2$	$1 \le i \le m$
$i \equiv 2(mod\ 3)$		$\sigma(x_i) = 0.3$	$1 \le i \le m$
$i \equiv 0(mod\ 3)$		$\sigma(x_i) = 0.1$	$1 \le i \le m$
$m \equiv 0,2(mod\ 3)$	$j \equiv 1,2(mod\ 3)$	$\sigma\left({y_j}^1\right) = 0.1$	$1 \le j \le n$
	$j \equiv 0(mod\ 3)$	$\sigma\left({y_j}^1\right) = 0.3$	$1 \le j \le n$
	$j \equiv 1,2(mod\ 3)$	$\sigma\left({y_j}^2\right) = 0.2$	$1 \le j \le n$
	$j \equiv 0(mod\ 3)$	$\sigma\left({y_j}^2\right) = 0.3$	$1 \le j \le n$
$m \equiv 1(mod\ 3)$	$j \equiv 0,1(mod\ 3)$	$\sigma\left({y_j}^1\right) = 0.1$	$1 \le j \le n$
	$j \equiv 2(mod\ 3)$	$\sigma\left({y_j}^1\right) = 0.3$	$1 \le j \le n$
	$j \equiv 0,2(mod\ 3)$	$\sigma\left({y_j}^2\right) = 0.2$	$1 \le j \le n$
	$j \equiv 1(mod\ 3)$	$\sigma\left({y_j}^2\right) = 0.3$	$1 \le j \le n$

Table 3.18.2						
Nature of m and n	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$m \equiv 0(mod\ 3)$ $n \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 0(mod\ 3)$ $n \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$
$m \equiv 0(mod\ 3)$ $n \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$
$m \equiv 1(mod\ 3)$ $n \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$
$m \equiv 1(mod\ 3)$ $n \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 1(mod\ 3)$ $n \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$
$m \equiv 2(mod\ 3)$ $n \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$
$m \equiv 2(mod\ 3)$ $n \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3} - 1$
$m \equiv 2(mod\ 3)$ $n \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$

From the above table 3.18.2 it is clear that for $i, j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \neq j$ the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1.
Hence it is concluded that the tree ${\mathcal{T}_{19}}^4$ is fuzzy quotient-3 cordial. □

Theorem 3.19: The tree ${\mathcal{T}_{21}}^5$ is fuzzy quotient-3 cordial.
Proof:
Consider the tree ${\mathcal{T}_{21}}^5$.
$V\left({\mathcal{T}_{21}}^5\right) = \{v_h : 1 \leq h \leq 5\} \cup \{x_i : 1 \leq i \leq m\} \cup \{x\} \cup \{y_j : 1 \leq j \leq n\} \cup \{z_k : 1 \leq k \leq l\}$.
$E\left({\mathcal{T}_{21}}^5\right) = \{v_h v_{h+1} : 1 \leq h \leq 4\} \cup \{v_2 x_i : 1 \leq i \leq m\} \cup \{v_3 x\} \cup \{x y_j : 1 \leq j \leq n\}$
$\cup \{v_4 z_k : 1 \leq k \leq l\}$.
$|V| = m + n + l + 6$ and $|E| = m + n + l + 5$.
We define $\sigma : V\left({\mathcal{T}_{63}}^5\right) \rightarrow [0,1]$ by $\sigma(v) = \frac{r}{10}, r \in Z_4 - \{0\}$.
$\sigma(x) = 0.1$, $\sigma(v_1) = 0.2$, $\sigma(v_2) = 0.3$, $\sigma(v_3) = 0.1$, $\sigma(v_4) = 0.3$, $\sigma(v_5) = 0.2$ and the values of $\sigma(x_i), 1 \leq i \leq m, \sigma(y_j), 1 \leq j \leq n, \sigma(z_k), 1 \leq k \leq l$ are tabulated below.

Table 3.19.1			
$h \equiv 3,4(mod\ 5)$		$\sigma(v_h) = 0.1$	$1 \leq h \leq 5$
$h \equiv 0,1(mod\ 5)$		$\sigma(v_h) = 0.2$	$1 \leq h \leq 5$
$h \equiv 2(mod\ 5)$		$\sigma(v_h) = 0.3$	$1 \leq h \leq 5$
$m \equiv 0,1,2(mod\ 3)$	$i \equiv 0(mod\ 3)$	$\sigma(x_i) = 0.1$	$1 \leq i \leq m$
	$i \equiv 2(mod\ 3)$	$\sigma(x_i) = 0.2$	$1 \leq i \leq m$
	$i \equiv 1(mod\ 3)$	$\sigma(x_i) = 0.3$	$1 \leq i \leq m$
$n \equiv 0,1(mod\ 3)$	$j \equiv 1(mod\ 3)$	$\sigma(y_j) = 0.1$	$1 \leq j \leq n$
	$j \equiv 2(mod\ 3)$	$\sigma(y_j) = 0.2$	$1 \leq j \leq n$
	$j \equiv 0(mod\ 3)$	$\sigma(y_j) = 0.3$	$1 \leq j \leq n$
$n \equiv 2(mod\ 3)$	$j \equiv 1(mod\ 3)$	$\sigma(y_j) = 0.1$	$1 \leq j \leq n$
	$j \equiv 0(mod\ 3)$	$\sigma(y_j) = 0.2$	$1 \leq j \leq n$

	$j \equiv 2(mod\ 3)$	$\sigma(y_j) = 0.3$	$1 \le j \le n$
$m \equiv 0(mod\ 3)$ $n \equiv 0,1(mod\ 3)$	$k \equiv 0(mod\ 3)$	$\sigma(z_k) = 0.1$	$1 \le k \le l$
	$k \equiv 2(mod\ 3)$	$\sigma(z_k) = 0.2$	$1 \le k \le l$
	$k \equiv 1(mod\ 3)$	$\sigma(z_k) = 0.3$	$1 \le k \le l$
$m \equiv 0(mod\ 3)$ $n \equiv 2(mod\ 3)$	$k \equiv 2(mod\ 3)$	$\sigma(z_k) = 0.1$	$1 \le k \le l$
	$k \equiv 0(mod\ 3)$	$\sigma(z_k) = 0.2$	$1 \le k \le l$
	$k \equiv 1(mod\ 3)$	$\sigma(z_k) = 0.3$	$1 \le k \le l$
$m \equiv 1(mod\ 3)$ $n \equiv 0,2(mod\ 3)$	$k \equiv 1(mod\ 3)$	$\sigma(z_k) = 0.1$	$1 \le k \le l$
	$k \equiv 2(mod\ 3)$	$\sigma(z_k) = 0.2$	$1 \le k \le l$
	$k \equiv 0(mod\ 3)$	$\sigma(z_k) = 0.3$	$1 \le k \le l$
$m \equiv 1(mod\ 3)$ $n \equiv 1(mod\ 3)$	$k \equiv 2(mod\ 3)$	$\sigma(z_k) = 0.1$	$1 \le k \le l$
	$k \equiv 1(mod\ 3)$	$\sigma(z_k) = 0.2$	$1 \le k \le l$
	$k \equiv 0(mod\ 3)$	$\sigma(z_k) = 0.3$	$1 \le k \le l$
$m \equiv 2(mod\ 3)$ $n \equiv 1(mod\ 3)$	$k \equiv 1(mod\ 3)$	$\sigma(z_k) = 0.1$	$1 \le k \le l$
	$k \equiv 0(mod\ 3)$	$\sigma(z_k) = 0.2$	$1 \le k \le l$
	$k \equiv 2(mod\ 3)$	$\sigma(z_k) = 0.3$	$1 \le k \le l$
$m \equiv 2(mod\ 3)$ $n \equiv 0,2(mod\ 3)$	$k \equiv 1(mod\ 3)$	$\sigma(z_k) = 0.1$	$1 \le k \le l$
	$k \equiv 2(mod\ 3)$	$\sigma(z_k) = 0.2$	$1 \le k \le l$
	$k \equiv 0(mod\ 3)$	$\sigma(z_k) = 0.3$	$1 \le k \le l$

Table 3.19.2

Nature of m, n *and* l	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$m \equiv 0(mod\ 3)$ $n \equiv 0(mod\ 3)$ $l \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$
$m \equiv 0(mod\ 3)$ $n \equiv 0(mod\ 3)$ $l \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 0(mod\ 3)$ $n \equiv 0(mod\ 3)$ $l \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$
$m \equiv 0(mod\ 3)$ $n \equiv 1(mod\ 3)$ $l \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 0(mod\ 3)$ $n \equiv 1(mod\ 3)$ $l \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$
$m \equiv 0(mod\ 3)$ $n \equiv 1(mod\ 3)$ $l \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$
$m \equiv 0(mod\ 3)$ $n \equiv 2(mod\ 3)$ $l \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$
$m \equiv 0(mod\ 3)$ $n \equiv 2(mod\ 3)$ $l \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$
$m \equiv 0(mod\ 3)$ $n \equiv 2(mod\ 3)$ $l \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 1(mod\ 3)$ $n \equiv 0(mod\ 3)$ $l \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 1(mod\ 3)$ $n \equiv 0(mod\ 3)$ $l \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3} - 1$

$m \equiv 1(mod\ 3)$ $n \equiv 0(mod\ 3)$ $l \equiv 2(mod\ 3)$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}-1$
$m \equiv 1(mod\ 3)$ $n \equiv 1(mod\ 3)$ $l \equiv 0(mod\ 3)$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}$
$m \equiv 1(mod\ 3)$ $n \equiv 1(mod\ 3)$ $l \equiv 1(mod\ 3)$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}-1$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$
$m \equiv 1(mod\ 3)$ $n \equiv 1(mod\ 3)$ $l \equiv 2(mod\ 3)$	$\frac{\|V\|-1}{3}+1$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$
$m \equiv 1(mod\ 3)$ $n \equiv 2(mod\ 3)$ $l \equiv 0(mod\ 3)$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}-1$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$
$m \equiv 1(mod\ 3)$ $n \equiv 2(mod\ 3)$ $l \equiv 1(mod\ 3)$	$\frac{\|V\|-1}{3}+1$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$
$m \equiv 1(mod\ 3)$ $n \equiv 2(mod\ 3)$ $l \equiv 2(mod\ 3)$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$
$m \equiv 2(mod\ 3)$ $n \equiv 0(mod\ 3)$ $l \equiv 0(mod\ 3)$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$
$m \equiv 2(mod\ 3)$ $n \equiv 0(mod\ 3)$ $l \equiv 1(mod\ 3)$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}-1$
$m \equiv 2(mod\ 3)$ $n \equiv 0(mod\ 3)$ $l \equiv 2(mod\ 3)$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}+1$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$
$m \equiv 2(mod\ 3)$ $n \equiv 1(mod\ 3)$ $l \equiv 0(mod\ 3)$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}-1$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$
$m \equiv 2(mod\ 3)$ $n \equiv 1(mod\ 3)$ $l \equiv 1(mod\ 3)$	$\frac{\|V\|-1}{3}+1$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$
$m \equiv 2(mod\ 3)$ $n \equiv 1(mod\ 3)$ $l \equiv 2(mod\ 3)$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$
$m \equiv 2(mod\ 3)$ $n \equiv 2(mod\ 3)$ $l \equiv 0(mod\ 3)$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}+1$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$
$m \equiv 2(mod\ 3)$ $n \equiv 2(mod\ 3)$ $l \equiv 1(mod\ 3)$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}-1$
$m \equiv 2(mod\ 3)$ $n \equiv 2(mod\ 3)$ $l \equiv 2(mod\ 3)$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}-1$

From the above tables 3.19.2 it is clear that for $i, j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \neq j$ the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1.

Hence it is concluded that the tree ${\mathcal{T}_{21}}^5$ is fuzzy quotient-3 cordial.□

Theorem 3.20: The tree ${\mathcal{T}_{22}}^{5}$ is fuzzy quotient-3 cordial.

Proof:

Consider the tree ${\mathcal{T}_{22}}^{5}$.

$V\left({\mathcal{T}_{22}}^{5}\right) = \{v_h : 1 \le h \le 5\} \cup \{x_i : 1 \le i \le m\} \cup \{{y_k}^1 : 1 \le j \le n\} \cup \{{y_k}^2, 1 \le j \le n\}$
$\cup \{z_k : 1 \le k \le l\}$.

$E\left({\mathcal{T}_{22}}^{5}\right) = \{v_h v_{h+1} : 1 \le h \le 4\} \cup \{v_2 x_i : 1 \le i \le m\} \cup \left\{v_3 {y_j}^1 : 1 \le j \le n\right\}$
$\cup \{{y_j}^1 {y_j}^2 : 1 \le j \le n\} \cup \{v_4 z_k : 1 \le k \le l\}$.

$|V| = m + 2n + l + 5$ and $|E| = m + 2n + l + 4$.

We define $\sigma : V\left({\mathcal{T}_{64}}^{5}\right) \to [0,1]$ by $\sigma(v) = \frac{r}{10}$, $r \in Z_4 - \{0\}$.

$\sigma(v_1) = 0.2$, $\sigma(v_2) = 0.3$, $\sigma(v_3) = 0.1$, $\sigma(v_4) = 0.1$, $\sigma(v_5) = 0.2$ and

the values of $\sigma(x_i), 1 \le i \le m, \sigma(y_j), 1 \le j \le n, \sigma({z_k}^t), 1 \le k \le l, t = 1,2$ are tabulated below.

Table 3.20.1

	$i \equiv 2(mod\ 3)$	$\sigma(x_i) = 0.1$	$1 \le i \le m$
	$i \equiv 0(mod\ 3)$	$\sigma(x_i) = 0.2$	$1 \le i \le m$
	$i \equiv 1(mod\ 3)$	$\sigma(x_i) = 0.3$	$1 \le i \le m$
$m \equiv 0,1(mod\ 3)$	$j \equiv 2(mod\ 3)$	$\sigma(y_j) = 0.1$	$1 \le j \le n$
	$j \equiv 0(mod\ 3)$	$\sigma(y_j) = 0.2$	$1 \le j \le n$
	$j \equiv 1(mod\ 3)$	$\sigma(y_j) = 0.3$	$1 \le j \le n$
$m \equiv 2(mod\ 3)$	$j \equiv 0(mod\ 3)$	$\sigma(y_j) = 0.1$	$1 \le j \le n$
	$j \equiv 2(mod\ 3)$	$\sigma(y_j) = 0.2$	$1 \le j \le n$
	$j \equiv 1(mod\ 3)$	$\sigma(y_j) = 0.3$	$1 \le j \le n$
$m \equiv 0(mod\ 3)$ $n \equiv 0(mod\ 3)$	$k \equiv 0,1(mod\ 3)$	$\sigma({z_k}^1) = 0.1$	$1 \le k \le l$
	$k \equiv 2(mod\ 3)$	$\sigma({z_k}^1) = 0.3$	$1 \le k \le l$
	$k \equiv 0,2(mod\ 3)$	$\sigma({z_k}^2) = 0.2$	$1 \le k \le l$
	$k \equiv 1(mod\ 3)$	$\sigma({z_k}^2) = 0.3$	$1 \le k \le l$
$m \equiv 0(mod\ 3)$ $n \equiv 1(mod\ 3)$	$k \equiv 1,2(mod\ 3)$	$\sigma({z_k}^1) = 0.1$	$1 \le k \le l$
	$k \equiv 0(mod\ 3)$	$\sigma({z_k}^1) = 0.3$	$1 \le k \le l$
	$k \equiv 0,2(mod\ 3)$	$\sigma({z_k}^2) = 0.2$	$1 \le k \le l$
	$k \equiv 1(mod\ 3)$	$\sigma({z_k}^2) = 0.3$	$1 \le k \le l$
$m \equiv 0(mod\ 3)$ $n \equiv 2(mod\ 3)$	$k \equiv 0,2(mod\ 3)$	$\sigma({z_k}^1) = 0.1$	$1 \le k \le l$
	$k \equiv 1(mod\ 3)$	$\sigma({z_k}^1) = 0.3$	$1 \le k \le l$
	$k \equiv 1,2(mod\ 3)$	$\sigma({z_k}^2) = 0.2$	$1 \le k \le l$
	$k \equiv 0(mod\ 3)$	$\sigma({z_k}^2) = 0.3$	$1 \le k \le l$
$m \equiv 1(mod\ 3)$ $n \equiv 0(mod\ 3)$	$k \equiv 0,2(mod\ 3)$	$\sigma({z_k}^1) = 0.1$	$1 \le k \le l$
	$k \equiv 1(mod\ 3)$	$\sigma({z_k}^1) = 0.3$	$1 \le k \le l$
	$k \equiv 1(mod\ 3)$	$\sigma({z_k}^2) = 0.2$	$1 \le k \le l$
	$k \equiv 0,2(mod\ 3)$	$\sigma({z_k}^2) = 0.3$	$1 \le k \le l$
$m \equiv 1(mod\ 3)$ $n \equiv 1(mod\ 3)$	$k \equiv 1,2(mod\ 3)$	$\sigma({z_k}^1) = 0.1$	$1 \le k \le l$
	$k \equiv 0(mod\ 3)$	$\sigma({z_k}^1) = 0.3$	$1 \le k \le l$
	$k \equiv 1(mod\ 3)$	$\sigma({z_k}^2) = 0.2$	$1 \le k \le l$
	$k \equiv 0,2(mod\ 3)$	$\sigma({z_k}^2) = 0.3$	$1 \le k \le l$
$m \equiv 1(mod\ 3)$ $n \equiv 2(mod\ 3)$	$k \equiv 0,2(mod\ 3)$	$\sigma({z_k}^1) = 0.1$	$1 \le k \le l$
	$k \equiv 1(mod\ 3)$	$\sigma({z_k}^1) = 0.3$	$1 \le k \le l$
	$k \equiv 1,2(mod\ 3)$	$\sigma({z_k}^2) = 0.2$	$1 \le k \le l$
	$k \equiv 0(mod\ 3)$	$\sigma({z_k}^2) = 0.3$	$1 \le k \le l$
$m \equiv 2(mod\ 3)$ $n \equiv 0(mod\ 3)$	$k \equiv 0,2(mod\ 3)$	$\sigma({z_k}^1) = 0.1$	$1 \le k \le l$
	$k \equiv 1(mod\ 3)$	$\sigma({z_k}^1) = 0.3$	$1 \le k \le l$
	$k \equiv 1(mod\ 3)$	$\sigma({z_k}^2) = 0.2$	$1 \le k \le l$
	$k \equiv 0,2(mod\ 3)$	$\sigma({z_k}^2) = 0.3$	$1 \le k \le l$
$m \equiv 2(mod\ 3)$ $n \equiv 1(mod\ 3)$	$k \equiv 0,1(mod\ 3)$	$\sigma({z_k}^1) = 0.1$	$1 \le k \le l$
	$k \equiv 2(mod\ 3)$	$\sigma({z_k}^1) = 0.3$	$1 \le k \le l$
	$k \equiv 1,2(mod\ 3)$	$\sigma({z_k}^2) = 0.2$	$1 \le k \le l$

$m \equiv 2(mod\ 3)$ $n \equiv 2(mod\ 3)$	$k \equiv 0(mod\ 3)$	$\sigma({z_k}^2) = 0.3$	$1 \le k \le l$
	$k \equiv 1,2(mod\ 3)$	$\sigma({z_k}^1) = 0.1$	$1 \le k \le l$
	$k \equiv 0(mod\ 3)$	$\sigma({z_k}^1) = 0.3$	$1 \le k \le l$
	$k \equiv 0,2(mod\ 3)$	$\sigma({z_k}^2) = 0.2$	$1 \le k \le l$
	$k \equiv 1(mod\ 3)$	$\sigma({z_k}^2) = 0.3$	$1 \le k \le l$

Table 3.20.2

Nature of m, n *and* l	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$m \equiv 0(mod\ 3)$ $n \equiv 0(mod\ 3)$ $l \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$
$m \equiv 0(mod\ 3)$ $n \equiv 0(mod\ 3)$ $l \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 0(mod\ 3)$ $n \equiv 0(mod\ 3)$ $l \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$
$m \equiv 0(mod\ 3)$ $n \equiv 1(mod\ 3)$ $l \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$
$m \equiv 0(mod\ 3)$ $n \equiv 1(mod\ 3)$ $l \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3} - 1$
$m \equiv 0(mod\ 3)$ $n \equiv 1(mod\ 3)$ $l \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 0(mod\ 3)$ $n \equiv 2(mod\ 3)$ $l \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 0(mod\ 3)$ $n \equiv 2(mod\ 3)$ $l \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$
$m \equiv 0(mod\ 3)$ $n \equiv 2(mod\ 3)$ $l \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$
$m \equiv 1(mod\ 3)$ $n \equiv 0(mod\ 3)$ $l \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$
$m \equiv 1(mod\ 3)$ $n \equiv 0(mod\ 3)$ $l \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$
$m \equiv 1(mod\ 3)$ $n \equiv 0(mod\ 3)$ $l \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 1(mod\ 3)$ $n \equiv 1(mod\ 3)$ $l \equiv 0(mod\ 3)$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3} + 1$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$	$\frac{\lvert V\rvert - 1}{3}$
$m \equiv 1(mod\ 3)$ $n \equiv 1(mod\ 3)$ $l \equiv 1(mod\ 3)$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3} - 1$	$\frac{\lvert V\rvert}{3}$	$\frac{\lvert V\rvert}{3}$
$m \equiv 1(mod\ 3)$ $n \equiv 1(mod\ 3)$ $l \equiv 2(mod\ 3)$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3} - 1$	$\frac{\lvert V\rvert + 1}{3}$

$m \equiv 1(mod\ 3)$ $n \equiv 2(mod\ 3)$ $l \equiv 0(mod\ 3)$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}$
$m \equiv 1(mod\ 3)$ $n \equiv 2(mod\ 3)$ $l \equiv 1(mod\ 3)$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}+1$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$
$m \equiv 1(mod\ 3)$ $n \equiv 2(mod\ 3)$ $l \equiv 2(mod\ 3)$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}-1$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$
$m \equiv 2(mod\ 3)$ $n \equiv 0(mod\ 3)$ $l \equiv 0(mod\ 3)$	$\frac{\|V\|-1}{3}+1$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$
$m \equiv 2(mod\ 3)$ $n \equiv 0(mod\ 3)$ $l \equiv 1(mod\ 3)$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}-1$
$m \equiv 2(mod\ 3)$ $n \equiv 0(mod\ 3)$ $l \equiv 2(mod\ 3)$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}-1$
$m \equiv 2(mod\ 3)$ $n \equiv 1(mod\ 3)$ $l \equiv 0(mod\ 3)$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}-1$
$m \equiv 2(mod\ 3)$ $n \equiv 1(mod\ 3)$ $l \equiv 1(mod\ 3)$	$\frac{\|V\|-1}{3}+1$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$
$m \equiv 2(mod\ 3)$ $n \equiv 1(mod\ 3)$ $l \equiv 2(mod\ 3)$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}-1$
$m \equiv 2(mod\ 3)$ $n \equiv 2(mod\ 3)$ $l \equiv 0(mod\ 3)$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}$	$\frac{\|V\|}{3}-1$
$m \equiv 2(mod\ 3)$ $n \equiv 2(mod\ 3)$ $l \equiv 1(mod\ 3)$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}$	$\frac{\|V\|+1}{3}-1$	$\frac{\|V\|+1}{3}-1$
$m \equiv 2(mod\ 3)$ $n \equiv 2(mod\ 3)$ $l \equiv 2(mod\ 3)$	$\frac{\|V\|-1}{3}+1$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$	$\frac{\|V\|-1}{3}$

From the above tables 3.20.2 it is clear that for $i,j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}, i \neq j$ the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1 and the number of edges labeled with i and the number of edges labeled with j differ by at most 1. Hence it is concluded that the tree ${\mathcal{T}_{22}}^5$is fuzzy quotient-3 cordial.□

4. CONCLUSION

In this paper we proved that some trees of diameter 2, 3 and 4 are fuzzy quotient -3 Cordial. and Also the existence of Fuzzy quotient-3 cordial labeling of trees of diameter 5 is proved and to be published in next journal.

5. REFERENCES

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