SOFT b-CONTINUOUS IN SOFT TRITOPOLOGICAL SPACE

Bharathi. S[1], Devi. T[2], Poongodi. D[3]

[1]Department of Mathematics,Bharathiar University PG Extension and ResearchCenter,Perundurai,Erode,Tamilnadu,India.
*[*2]Department of Mathematics, Builders Engineering College,Kangayam,Tiruppur,Tamilnadu,India.*
[3]Department of Mathematics,Bharathiar University PG Extension and ResearchCenter,Perundurai,Erode,Tamilnadu,India
*[*1]Poon81godi@gmail.com*

ABSTRACT:*In this paper introduce the* $(1,2,3)^*$*soft b continuous function in Soft Tritopological Space. Also investigating some new characterizations of* $(1,2,3)^*$*soft semi continuous,* $(1,2,3)^*$*soft pre continuous,* $(1,2,3)^*$*soft continuous,* $(1,2,3)^*$ *soft* β *continuous in Soft Tritopological Space.*

KEYWORDS: **$(1,2,3)^*$soft b continuous function,$(1,2,3)^*$soft semi continuous, $(1,2,3)^*$soft pre continuous $(1,2,3)^*$ soft β continuous**

1. INTRODUCTION

In the year 1999,Molodtsov [4]proposed the theory of soft sets with new dimension to explain the practical difficulties in engineering physics,computerscience,economics,social science and medical science.MetinAkdag and AlkanOzkan[5] defined soft b open sets in soft topological space and investigates the properties of soft b continuity and soft b closed map.Revathi.N,Bageerathi.K[7] introduced the concept of $(1,2)^*$ soft b continuous and $(1,2)^*$soft b closed map.S.Muthuvel and R.Parimalazhagan [7] introduced the concept of b^* continuous functions in Topological space.AsmhanFlienHassan[1] introduced soft Tritopological spaces .Muhammad ShabirandMunazzaNaz[6] setforth soft topological spaces which are defined over an initial universe with a fixed set of criteria

2. PRELIMINARIES

Definition 2.1 [3]

A pair (F,A) is called a soft set over X, Where F is a mapping given by F:A → P(X).In other words ,a soft set over X is a parameterized family of subset of the universe X. For e∈ $A, F(e)$ may be considered as the set e-approximate elements of the soft set (F,A).

Definition 2.2[3]

For two soft sets (F,A) and (G,B) over a common universe X, we say that (F,A) is a soft subset of (G,B) ifi) A⊆B

ii) ∀e∈ A,F(e) $\subseteq$ G(e).

we write (F,A) $\subseteq$ (G,B).(F,A) is said to be a soft super set of (G,B).if (G,B) is a soft subset of (F,A) and is denoted by (F,A) $\supseteq$ (G,B).

Definition 2.3[3]

For two soft sets (F,A) and (G,B) over a common universe X, union of two soft sets of (F,A) and (G,B) is the soft (H,C),where C=A∪B and ∀ e ∈ C

$$H(e)=\begin{cases} F(e) & if\ e \in A-B \\ G(e) & if\ e \in B-A \\ F(e) \cup G(e) & if\ e \in A \cap B \end{cases}$$

We write (F,A) ∪(G,B)=(H,C)

Definition 2.4[3]

The intersection (H,C) of two soft sets (F,A) and (G,B) over a common universe X denoted by (F,A) ∩(G,B) is defined as C=A∩B and H(e)=F(e)∩G(e) ∀ e ∈ C

Definition 2.5 [3]

Let $\bar{X} \in S(X)$ power set of $\bar{X}$ is defined by $P(\bar{X}) = \{\bar{X}_\iota \subseteq \bar{X},\ i \in I\}$and its cardinality is defined by

$|P(\bar{X})| = 2^{\sum x \in E|F(x)|}$, where $|F(X)|$ is cardinality of F(x).

Definition 2.5 [3]

Let τ be the collection of soft sets over X ,then τ is called a soft topology on X if τ satisfies the following axioms

i)ϕ,$\bar{X}$ belongs to τ

ii) The union of any number of soft sets in τ belongs to τ

iii) The intersection of any two soft sets in τ belongs to τ

The triplet (X, τ ,E) is called a soft topological space over X.

Definition 2.6 [3]

A set X together with two different topologies is called bitopologicalspace .

It is denoted by (X, τ_1, τ_2).

Definition 2.7[2]

A set X together with three different topologies is called tritopologicalspace .

It is denoted by $(X, \tau_1, \tau_2, \tau_3)$.

Definition 2.10[2]

Let $(X, \tau_1, \tau_2, \tau_3)$ be a soft tritopological space and$^{and}(A, E) \subseteq \bar{X}$,Then (A,E) is called $(1,2,3)^*$ soft b-open set $(A, E) \subseteq \tau_{1,2,3} - \text{int}(\tau_{1,2,3} - cl(A, E)) \cup \tau_{1,2,3} - cl(\tau_{1,2,3} - \text{int}(A, E))$

Definition 2.11[2]

Let $(X, \tau_1, \tau_2, \tau_3)$ be a soft tritopological space and $(A, E) \subseteq \bar{X}$ .Then (A,E) is called $(1,2,3)^*$ soft b closed set if $\tau_{1,2,3} - int(\tau_{1,2,3} - cl(A, E)) \cup \tau_{1,2,3} - cl(\tau_{1,2,3} - int(A, E)) \subseteq (A, E)$

Definition 2.12[9]

A soft mapping $\tilde{f}: (\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E) \to (\tilde{Y}, \tilde{\sigma}_1, \tilde{\sigma}_2, K)$ is said to be $(1,2)^*$ soft continuous if the inverse image of every $\tilde{\sigma}_{1,2}$open set of $\tilde{Y}$ is a $\tilde{\tau}_{1,2}$ open set in $\tilde{X}$.

Definition 2.13[9]

A soft mapping $\tilde{f}: (\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E) \to (\tilde{Y}, \tilde{\sigma}_1, \tilde{\sigma}_2, K)$ is said to be $(1,2)^*$ soft b continuous if the inverse image of every $\tilde{\sigma}_{1,2}$open set of $\tilde{Y}$ is a $(1,2)^*$ soft b open set in $\tilde{X}$.

3. $(1, 2, 3)^*$SOFT b CONTINUOUS FUNCTION

Definition 3.1

A soft mapping $f: (X, \tau_1, \tau_2, \tau_3, E) \to (Y, \sigma_1, \sigma_2, \sigma_3, M)$ is said to be $(1,2,3)^*$ soft continuous if the inverse image of every $\sigma_{1,2,3}$open set of Y is a $\tau_{1,2,3}$ open set in X.

Definition 3.2

A soft mapping $f: (X, \tau_1, \tau_2, \tau_3, E) \to (Y, \sigma_1, \sigma_2, \sigma_3, M)$ is said to be $(1,2,3)^*$ soft b continuous (briefly sb continuous) the inverse image of every $\sigma_{1,2,3}$ open set of Y is a $(1,2,3)^*$ soft b open set in X.

Example 3.3

Let $X = Y = \{l, m\}$,$E = M = \{e_1, e_2\}$ and$\bar{X} = \bar{Y} = \{(e_1, \{l, m\}), (e_2, \{l, m\})\}$ .Thesoft subsets are

$$A_{E_1} = \{(e_1, \{l\}), (e_2, \{l\})\} \quad A_{E_9} = \{(e_1, \{X\}), (e_2, \{l\})\}$$

$$A_{E_2} = \{(e_1, \{l\}), (e_2, \{m\})\} \quad A_{E_{10}} = \{(e_1, \{X\}), (e_2, \{m\})\}$$

$$A_{E_3} = \{(e_1, \{l\}), (e_2, \{X\})\} \quad A_{E_{11}} = \{(e_1, \{X\}), (e_2, \{X\})\}$$

$$A_{E_4} = \{(e_1, \{l\}), (e_2, \{\emptyset\})\} \quad A_{E_{12}} = \{(e_1, \{X\}), (e_2, \{\emptyset\})\}$$

$$A_{E_5} = \{(e_1, \{m\}), (e_2, \{l\})\} A_{E_{13}} = \{(e_1, \{\emptyset\}), (e_2, \{l\})\}$$

$$A_{E_6} = \{(e_1, \{m\}), (e_2, \{m\})\} \quad A_{E_{14}} = \{(e_1, \{\emptyset\}), (e_2, \{m\})\}$$

$$A_{E_7} = \{(e_1, \{m\}), (e_2, \{X\})\} \quad A_{E_{15}} = \{(e_1, \{\emptyset\}), (e_2, \{X\})\}$$

$$A_{E_8} = \{(e_1, \{m\}), (e_2, \{\emptyset\})\} \quad A_{E_{16}} = \{(e_1, \{\emptyset\}), (e_2, \{\emptyset\})\}$$

Consider the soft tritopological spaces $(X, \tau_1, \tau_2, \tau_3, E)$ and $(Y, \sigma_1, \sigma_2, \sigma_3, M)$ where $\tau_1 = \{\bar{X}, \emptyset, A_{E_4}\}$,$\tau_2 = \{\bar{X}, \emptyset, A_{E_8}\}$and $\tau_3 = \{\bar{X}, \emptyset, A_{E_{12}}\}$.

Then $\tau_{1,2,3} = \{\bar{X}, \emptyset, A_{E_4}, A_{E_8}, A_{E_{12}}\}$ and $(1,2,3)^*$ SbO(X)= $\{\bar{X}, \emptyset, A_{E_1}, A_{E_2}, A_{E_3}, A_{E_4}, A_{E_5}, A_{E_6}, A_{E_7}, A_{E_8}, A_{E_9}, A_{E_{10}}\}$ .Let $\sigma_1 = \{Y, \emptyset, A_{E_1}, A_{E_4}\}$, $\sigma_2 = \{Y, \emptyset, A_{E_{12}}\}$ and $\sigma_3 = \{Y, \emptyset, A_{E_4}\}$ .Then $\sigma_{1,2,3} = \{Y, \emptyset, A_{E_1}, A_{E_4}, A_{E_9}, A_{E_{12}}\}$ .Define $f: X \to Y$ and $v: E \to M$ such that $f(l) = l, f(m) = m$ and $v(e_1) = e_1, v(e_2) = e_2$.Then the inverse image of every $\sigma_{1,2,3}$ open set of Y is a $(1,2,3)^*$ soft b open set in X. Hence $f: (X, \tau_1, \tau_2, \tau_3, E) \to (Y, \sigma_1, \sigma_2, \sigma_3, M)$ is said to be $(1,2,3)^*$ soft b continuous.

Proposition 3.4

Every $(1,2,3)^*$ soft continuous function is $(1,2,3)^*$ soft b continuous.

Proof: Let $f: (X, \tau_1, \tau_2, \tau_3, E) \to (Y, \sigma_1, \sigma_2, \sigma_3, M)$ is a $(1,2,3)^*$ soft continuous function and A_M is $\sigma_{1,2,3}$

open set in Y.Since f is $(1,2,3)^*$soft continuous,$f^{-1}(A_M)$ is a $\tau_{1,2,3}$ open set in X and every $\tau_{1,2,3}$ open

set in X is a $(1,2,3)^*$ soft b open set in X.Therefore f is $(1,2,3)^*$soft b continuous.

Remark 3.5

A $(1,2,3)^*$ soft b continuous function need not be $(1,2,3)^*$soft continuous is shown in the following example.

Example 3.6

Consider the soft mapping given in the Example3.3, $f: (X, \tau_1, \tau_2, \tau_3, E) \to (Y, \sigma_1, \sigma_2, \sigma_3, M)$ is a $(1,2,3)^*$ soft b continuous function. Take the $\sigma_{1,2,3}$ open sets A_{E_1}, A_{E_9} in Y.The inverse image of A_{E_1} and A_{E_9} are itself only. But A_{E_1} and A_{E_9} are not $\tau_{1,2,3}$ open sets in X.Hence $f: X \to Y$ is not $(1,2,3)^*$ soft continuous function.

Theorem 3.7

Let $f: (X, \tau_1, \tau_2, \tau_3, E) \to (Y, \sigma_1, \sigma_2, \sigma_3, M)$ be a soft mapping from a soft tritopological space X into a soft tritopological space Y.Then the following statements are equivalent.

i) f is$(1,2,3)^*$ soft b continuous

ii)The inverse image of every $\sigma_{1,2,3}$ closed set of Y is a $(1,2,3)^*$ soft b closed set in X

Proof.i)→ii) Let G_M be any $\sigma_{1,2,3}$ closed set of $Y$and ${G_M}^C$ is $\sigma_{1,2,3}$open set in Y.Since f is $(1,2,3)^*$ soft b continuous, $f^{-1}({G_M}^C)$ is $(1,2,3)^*$ soft b open set in X and $f^{-1}({G_M}^C) = (f^{-1}(G_M))^C$ .Therefore $f^{-1}(G_M)$ is $(1,2,3)^*$ soft b closed set in X.

ii)→ i)Let A_M be any $\sigma_{1,2,3}$ open set of Y.Then ${A_M}^C$ is $\sigma_{1,2,3}$ closed set in Y.Then by ii), $f^{-1}({A_M}^C)$ is $(1,2,3)^*$soft b closed set in X and $f^{-1}({A_M}^C) = (f^{-1}(A_M))^C$ .Theore fore $f^{-1}(A_M)$ is $(1,2,3)^*$ soft b open set in X.Hence f is $(1,2,3)^*$ soft b continuous.

Definition 3.8

A soft mapping $f: (X, \tau_1, \tau_2, \tau_3, E) \to (Y, \sigma_1, \sigma_2, \sigma_3, M)$ is said to be

i)$(1,2,3)^*$soft semi continuous(briefly $(1,2,3)^*$ ss continuous) if the inverse image of every $\sigma_{1,2,3}$ open set of Y is a $(1,2,3)^*$ soft semi open set in X.

ii) $(1,2,3)^*$soft pre continuous(briefly $(1,2,3)^*$ sp continuous) if the inverse image of every $\sigma_{1,2,3}$ open set of Y is a $(1,2,3)^*$ soft pre open set in X.

iii) $(1,2,3)^*$softβ continuous(briefly $(1,2,3)^*$ sβ continuous) if the inverse image of every $\sigma_{1,2,3}$ open set of Y is a $(1,2,3)^*$ soft β open set in X.

Preposition 3.9 Let $f: (X, \tau_1, \tau_2, \tau_3, E) \to (Y, \sigma_1, \sigma_2, \sigma_3, M)$ be a soft mapping from a soft tritopological space X into a soft tritopological space Y.Then the following hold.

i) Every $(1,2,3)^*$ soft semi continuous function is $(1,2,3)^*$ soft b continuous.

ii)Every $(1,2,3)^*$ soft pre continuous function is $(1,2,3)^*$ soft b continuous.

iii)Every $(1,2,3)^*$ soft b continuous function is $(1,2,3)^*$ softβ continuous.

Proof:i) Let A_u be $\sigma_{1,2,3}$ open set in Y .Since f is a $(1,2,3)^*$ soft semi continuous ,$f^{-1}(A_E)$ is $(1,2,3)^*$ soft semi open set in X. Since every $(1,2,3)^*$ soft semi open set is $(1,2,3)^*$ soft b open set ,$f^{-1}(A_E)$ is $(1,2,3)^*$ soft b open set in X .Hence f is a $(1,2,3)^*$ soft b continuous.

The proof of ii) and iii) are similar to i) .

Remark 3.10 The reverse implication of the Preposition 3.9 needs not be true is shown in the following examples.

Example 3.11 A $(1,2,3)^*$ soft b continuous function need not be $(1,2,3)^*$ soft semi continuous.

Let $X = Y = \{l, m\}, E = M = \{e_1, e_2\}$and $\overline{X} = \overline{Y} = \{(e_1, \{l, m\}), (e_2, \{l, m\})\}$. The possible soft subsets are as in Example 3.5.

Consider the soft tritopological space $(X, \tau_1, \tau_2, \tau_3, E)$ and $(Y, \sigma_1, \sigma_2, \sigma_3, M)$ where Let$\tau_1 = \{\overline{X}, \emptyset, A_{E_4}\}$, $\tau_2 = \{\overline{X}, \emptyset, A_{E_7}\}$ and $\tau_3 = \{\overline{X}, \emptyset\}$ .Then $\tau_{1,2,3}$ open sets $= \{\overline{X}, \emptyset, A_{E_4}, A_{E_7}\}$ and $(1,2,3)^*$ SbO(X)=$\{\overline{X}, \emptyset, A_{E_1}, A_{E_2}, A_{E_3}, A_{E_4}, A_{E_5}, A_{E_6}, A_{E_7}, A_{E_8}, A_{E_9}, A_{E_{10}}, A_{E_{12}}, A_{E_{13}}, A_{E_{14}}, A_{E_{15}}\}$. And $(1,2,3)^*$ SSO(X)= $\{\overline{X}, \emptyset, A_{E_7}, A_{E_8}\}$ .Let $\sigma_1 = \{\bar{X}, \emptyset, A_{E_4}\}$, $\sigma_2 = \{\bar{X}, \emptyset, A_{E_8}\}$ and $\sigma_3 = \{\bar{X}, \emptyset, A_{E_{12}}\}$.Then$\sigma_{1,2,3}$ open sets = $\{\bar{X}, \emptyset, A_{E_4}, A_{E_8}, A_{E_{12}}\}$.Define$f: X \to Y$ and $v: E \to M$ such that $f(l) = l, f(m) = m$ and $v(e_1) = e_1, v(e_2) = e_2$.Then $f: X \to Y$ is $(1,2,3)^*$ soft b continuous . But inverse image of every $\sigma_{1,2,3}$open sets are not $(1,2,3)^*$ soft semi open set in X.Here$f^{-1}(A_{E_4}) = A_{E_4}$ and $f^{-1}(A_{E_{12}}) = A_{E_{12}}$ which are not $(1,2,3)^*$ soft semi open set in X. Therefore $f: X \to Y$ is not $(1,2,3)^*$ soft semi continuous.

Example 3.12 A $(1,2,3)^*$ soft b continuous function need not be $(1,2,3)^*$ soft pre continuous.

Consider the soft mapping $f: X \to Y$ as in the Example 3.3, which is $(1,2,3)^*$ soft b continuous function but not $(1,2,3)^*$ soft pre continuous. The set of all $(1,2,3)^*$ soft pre open set are $(1,2,3)^*$SPO(X)== $\{\bar{X}, \emptyset, A_{E_4}, A_{E_8}, A_{E_9}, A_{E_{10}}, A_{E_{12}}\}$.But$f^{-1}(A_{E_1}) = A_{E_1}$which is not $(1,2,3)^*$ soft pre open set in X.Therefore $f: X \to Y$ is not $(1,2,3)^*$ soft precontinuous.

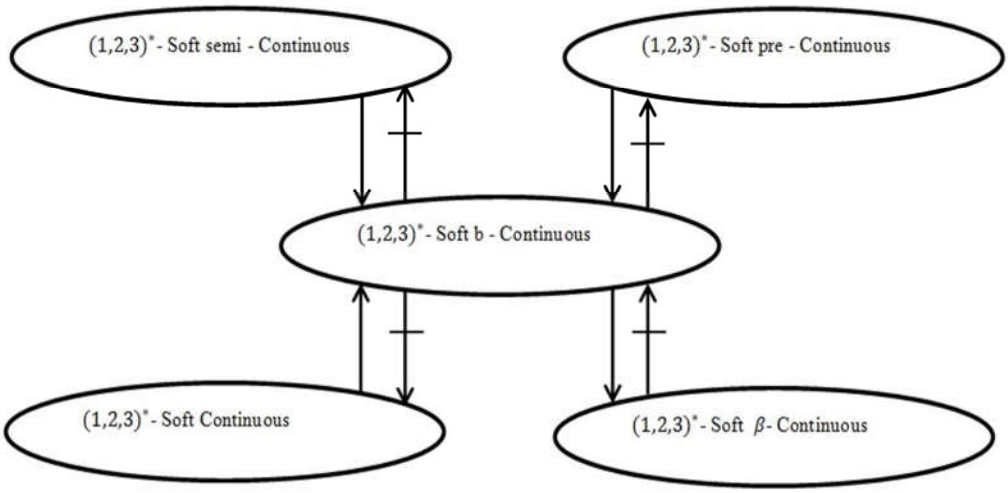
Example 3.16A$(1,2,3)^*$ soft β continuous function need not be $(1,2,3)^*$ soft b continuous.

Let$X = Y = \{l, m\}, E = M = \{e_1, e_2\}$and $\overline{X} = \overline{Y} = \{(e_1, \{l, m\}), (e_2, \{l, m\})\}$. The possible soft subsetsare as in Example 3.3.Consider the soft tritopological spaces $(X, \tau_1, \tau_2, \tau_3, E)$and $(Y, \sigma_1, \sigma_2, \sigma_3, M)$ where $\tau_1 = \{\bar{X}, \emptyset, A_{E_4}\}$,$\tau_2 = \{\bar{X}, \emptyset, A_{E_8}\}$ and $\tau_3 = \{\bar{X}, \emptyset, A_{E_{12}}\}$.Then $\tau_{1,2,3}$ open sets= $\{\bar{X}, \emptyset, A_{E_4}, A_{E_8}, A_{E_{12}}\}$, $(1,2,3)^*$ SbO(X)= $\{\bar{X}, \emptyset, A_{E_1}, A_{E_2}, A_{E_3}, A_{E_4}, A_{E_5}, A_{E_6}, A_{E_7}, A_{E_8}, A_{E_9}, A_{E_{10}}, A_{E_{12}},\}$ and $(1,2,3)^*$ SβO(X)=$\{\bar{X}, \emptyset, A_{E_1}, A_{E_2}, A_{E_3}, A_{E_4}, A_{E_5}, A_{E_6}, A_{E_7}, A_{E_8}, A_{E_9}, A_{E_{10}}, A_{E_{13}}, A_{E_{14}}\}$.
Let $\sigma_1 = \{\bar{Y}, \emptyset, A_{E_1}, A_{E_4}, A_{E_9}\}$,$\sigma_2 = \{\bar{Y}, \emptyset, A_{E_5}, A_{E_{13}}\}$ and $\sigma_3 = \{\bar{Y}, \emptyset, A_{E_9}, A_{E_{13}}\}$.$\sigma_{1,2,3}$open sets =$\{\bar{Y}, \emptyset, A_{E_1}, A_{E_4} A_{E_5}, A_{E_9}, A_{E_{13}}\}$.Define$f: X \to Y$ and $v: E \to M$ such that $f(l) = m, f(m) = l$and $v(e_1) = e_1, v(e_2) = e_2$. Then $f: X \to Y$ is $(1,2,3)^*$ soft β continuous.But inverse image of every $\sigma_{1,2,3}$ open sets are not$(1,2,3)^*$ soft b open set in$X$ because $f^{-1}(A_{E_{13}}) = A_{E_{14}}$ which is not $(1,2,3)^*$ soft b open set in X.Therefore $f: X \to Y$ is not $(1,2,3)^*$ soft b continuous.

Remark 3.17 From the above prepositions,examples and remark,we obtain the following figure,

Where $P \to Q$ indicates that $P's$ are $Q's$

$P \nrightarrow Q$Indicates that P need not be Q



4. CONCLUSION

This paper deals with the properties of $(1,2,3)^*$ soft b continuous, $(1,2,3)^*$soft semi continuous, $(1,2,3)^*$soft pre continuous ,$(1,2,3)^*$soft continuous, $(1,2,3)^*$ soft β continuous in Soft Tritopological Space and established the relations between them. Some new characterizations of these mappings and investigate preservation properties obtained through this paper. In future this paper will be basis for further applications of soft mapping in soft tritopological spaces.

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