MHD Flow Past an Exponentially Accelerated Infinite Isothermal Vertical Plate with Hall Current and Viscous Dissipation: FEM

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Abstract:

This work focused on, analyzing the effect of Hall current, Hartmann number, mass diffusion, thermal diffusion, rotation, thermal buoyancy force, concentration buoyancy force, radiation, viscous dissipation on the flow field of the MHD flow past an exponentially accelerated infinite isothermal vertical plate with uniform temperature. In view of it, the governed momentum, energy and diffusion non-linear partial differential equations are transformed into a system of non-linear dimension less partial differential equations with appropriate similarity variables. The resulting partial differential equations are solved under the initial and boundary conditions using finite element method. The significant physical parameters effects on primary and secondary fluid velocities, temperature, concentration distributions along boundary layer are expressed graphically and discussed. Validation of finite element solutions for heat and mass transfer coefficients are achieved via comparison with the previously published work as special cases of the present investigation and very good correlation obtained.

Keywords: MHD, Hall current, Viscous dissipation, FEM.

1. Introduction

Magneto hydrodynamics is the study of the dynamics of conducting fluids. Examples of such fluids include plasmas, liquid metals, and salt water or electrolytes MHD have applications in three different subject areas, such as astrophysical, geophysical and engineering problems. In addition, MHD free convection flows have significant applications in the field of stellar and planetary magneto spheres, aeronautical plasma flows and electronics. Recent advances in understanding the physics of flows and modeling the computational flow make tremendous contributions in chemical engineering.

The phenomenon of heat and mass transfer has been the object of extensive research due to its applications in science and technology. Such phenomenon is observed in buoyancy induced motions in the atmosphere, the bodies of water, the quasi-solid bodies such as earth, and so on. In nature and industrial applications, many transport processes exist where the heat and mass transfer takes place simultaneously as a result of combined effects of thermal diffusion and diffusion of chemical species. In addition, the phenomenon of heat mass transfer is also encountered in chemical processes industries such as food processing and polymer production. Considerable attention has been given to the unsteady free-convection flow of viscous incompressible and electrically conducting fluid in the presence of applied magnetic field in connection with the theory of fluid

motion in the liquid core of the Earth, meteorological, and oceanographic applications. Due to the gyration and drift of charged particles, the conductivity parallel to the electric field is reduced and the current is induced in the direction normal to both electric and magnetic fields. This phenomenon is known as the "Hall effect". The applications of the effect of Hall current on the fluid flow with variable concentration have been seen in MHD power generators. The Hall Effect is due merely to the sideways magnetic force on the drifting free charges. This effect on the fluid flow with variable concentration has a lot of applications in magnetohydrodynamic (MHD) power generators, general astrophysical and meteorological studies, and it can be taken into account within the range of magneto hydro dynamical approximations.

Viscous dissipation occurs in natural convection in various devices. Such dissipation effects may also be present in stronger gravitational fields and in process wherein the scale of the process is very large, e.g., on larger planets, in large masses of gas in space, and in geological processes in fluids internal to various bodies. With viscous dissipative heat included in the energy equation, S Sivaiah, and R Srinivasa Raju [1] presented the Finite element solution of heat and mass transfer flow with Hall current, heat source, and viscous dissipation. J A Rao and S Shivaiah [2] have studied chemical reaction effects on unsteady MHD flow past semi- infinite vertical porous plate with viscous dissipation. Siva Reddy Sheri and R Srinivasa Raju [3] investigated Transient MHD free convective flow past an infinite vertical plate embedded in a porous medium with viscous dissipation. Gebhart [4] studied the importance of viscous dissipative heat in free convective flows on a vertical surface subject to isothermal and uniform flux surface conditions. A finite-difference solution of the transient free convective flow of a viscous dissipative fluid past an infinite vertical plate, taking into account viscous dissipative heat, was studied by Soundalgekar et al. [5] using an implicit finite-difference method. Transient free convection flows of viscous dissipative fluid past a semi-infinite vertical plate, using an implicit finite-difference scheme, were studied by Soundalgekar et al. [6]. Ganesan and Palani [7] studied transient unsteady viscous dissipative fluid flow past a semi-infinite isothermal inclined plate by finite-difference scheme. Effect of viscous dissipation and pressure stress work in natural convection along a vertical isothermal plate was studied by Pantokratoras [8]. They solved the equations with the direct numerical solution of the boundary-layer equations without any approximation. The steady laminar boundary-layer flow along a vertical stationary heated plate is studied taking into account the viscous dissipation of the fluid studied by Pantokratoras [9]. The results are obtained with the numerical solution of the boundary-layer equations. Magyari and Rees [10] studied the effect of viscous dissipation on the Darcy free convection boundary-layer flow over a vertical plate with exponential temperature distribution in a porous medium. They studied in detail the heat transfer characteristics of all this self-similar free ion flows analytically and numerically. Some recent MHD convection papers were focused on the extended effects such as viscous dissipation and Joule heating. These effects are oriented to be important for large variations of gravitational force, operating at high rotational speeds and strong magnetic field. Takher and Soundalgekar [11] studied the effect of viscous dissipation and Joule heating on the MHD free convection flow past a vertical plate. El-Amin [12] studied the viscous dissipation and Joule heating effects on the MHD forced convection flow from a horizontal circular cylinder embedded in a fluid saturated porous medium. The study of the Hall effect on the MHD free convection flow along a vertical plate was analyzed by Pop and Watanabe [13]. Megahed et al. [14] carried out a similarity analysis in the MHD flow with heat and mass transfer along a vertical plate in the presence of a strong non uniform magnetic field and the Hall currents are taken into account. Sheela-Francisca J and Tso C P [15] studied viscous dissipation effects on parallel plates with constant heat flux boundary conditions. R. Muthucumaraswamy† and

K. M. A. Prema [16] analyzed Hall Effects on Flow past an Exponentially Accelerated Infinite Isothermal Vertical Plate with Mass Diffusion. G Palani and Kwang Yong Kim [17] discussed Joule Heating and Viscous Dissipation Effects on MHD Flow Past a Semi-Infinite Inclined Plate with Variable Surface Temperature. Partial differential equations which are the basic equations governing the fluid flow field, transformed by using similarity transformations, and the resultant dimensionless equations are solved using finite element method. The effects of parameter on fluid velocity, temperature and concentration have been shown graphically.

2. Formulation of the problem

Consider the transient MHD flow of a viscous incompressible fluid past an exponentially accelerated infinite isothermal vertical plate with uniform temperature, in the presence of thermal radiation. To explain geometrically three dimensional coordinate system has been used. Here $x-axis$ is chosen in the vertically upward direction to the plate and $y-axis$ is considered as normal to the plate. In a direction parallel to $y-axis$, a uniform sloping magnetic field B_0 is applied. The plate and fluid rotate in unison with uniform angular velocity Ω' about $y-axis$. Initially, (i.e., at time $t' \le 0$) the fluid, plate are at rest and they are at equal temperature T'_∞ also, within the fluid at all the points concentration C'_∞. At time $t' > 0$, exponentially accelerated plate with prescribed velocity $u = u_0 \exp(a't')$, (where u_0 is the velocity of the plate) in its own plane, at the same time the fluid temperature and level of concentration near the plate are raised linearly with time t. The geometry of the problem is presented in Figure 1. Since plate is of infinite extent in x and z directions and is electrically non-conducting, all physical quantities except pressure depend on y and t' only. Then under the usual Boussinesq's approximation the unsteady flow equations are momentum equation, energy equation and diffusion equation respectively.

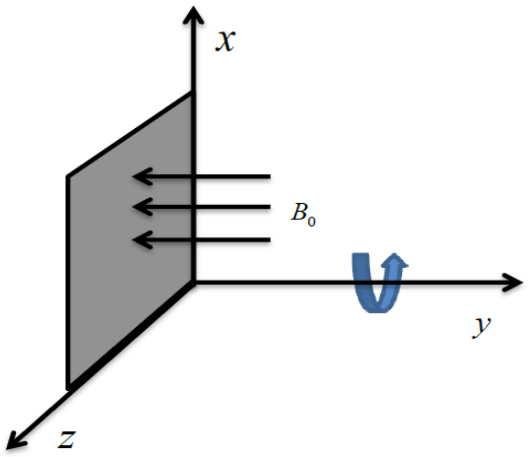

Figure 1. Geometry of the problem.

Equation of Momentum:

$$\frac{\partial u}{\partial t'} - 2\Omega' v_0 = \nu \frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho}\frac{\partial p}{\partial x} + g + \frac{B_0}{\rho} j_z \qquad (1)$$

$$\frac{\partial v_0}{\partial t'} + 2\Omega' u = \nu \frac{\partial^2 v_0}{\partial y^2} - \frac{B_0}{\rho} j_x \tag{2}$$

Equation of Energy:

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y^2} - \frac{\partial q_r}{\partial y} + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

Equation of Diffusion:

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} \tag{4}$$

Where T' is the fluid temperature, C' is the concentration, t' is the time, D is the diffusion term. Since there is no large velocity gradient here, the viscous term in equation (1) vanishes for small and hence for the outer flow, beside there is no magnetic field along x -direction gradient, so we have

$$0 = -\frac{\partial p}{\partial x} - \rho_\infty g \tag{5}$$

By eliminating the pressure term from equations (1) and (5), we obtain

$$\frac{\partial u}{\partial t'} - 2\Omega' v_0 = \nu \frac{\partial^2 u}{\partial y^2} + \left(\rho_\infty - \rho\right) g + \frac{B_0}{\rho} j_z \tag{6}$$

The Boussinesq's approximation gives

$$\rho_\infty - \rho = \rho_\infty \beta\left(T' - T'_\infty\right) + \rho_\infty \beta^*\left(C' - C'_\infty\right) \tag{7}$$

On using (7) in the equation (6) and noting that ρ_∞ is approximately equal to 1, the momentum equation reduces to

$$\frac{\partial u}{\partial t'} - 2\Omega' v_0 = \nu \frac{\partial^2 u}{\partial y^2} + \frac{B_0}{\rho} j_z + g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) \tag{8}$$

Where ρ is the density, ν the kinematic viscosity, u *and* v_0 are fluid velocity components, g the acceleration due to gravity, C_p the specific heat at constant pressure, q_r the radiative heat flux in the y-direction, k is thermal conductivity, β the volumetric coefficient of thermal expansion, β^* the volumetric coefficient of expansion with concentration.The generalized Ohm's law, on taking Hall current into account and neglecting ion-slip and thermoelectric effect, is

$$\vec{j} + \frac{\omega_e \tau_e}{B_o}\left(\vec{j} \times \vec{B}\right) = \sigma\left(\vec{E} + \vec{q} \times \vec{B}\right) \tag{9}$$

Where $\vec{j}$ is the current density vector, $\vec{q}$ the velocity vector, $\vec{B}$ the magnetic field vector, $\vec{E}$ the electric field vector, ω_e the cyclotron frequency, σ the electrical conductivity of the fluid and τ_e the collision time of electron.

The equation (9) gives

$$j_x - m j_z = \sigma v_0 B_0 \tag{10}$$

$$j_z + m j_x = -\sigma u B_0 \tag{11}$$

here $m = \omega_e \tau_e$ is the hall parameter. Solving equations (10) and (11) for j_x *and* j_z, we have

$$j_x = \frac{\sigma B_0}{1+m^2}\left(v_0 - mu\right) \tag{12}$$

$$j_z = \frac{\sigma B_0}{1+m^2}\left(u + mv_0\right) \tag{13}$$

On the use of equation (12) and (13), the momentum equations (8) and (2) become

$$\frac{\partial u}{\partial t'} = \nu \frac{\partial^2 u}{\partial y^2} + 2\Omega' v_0 - \frac{\sigma B_0^2 \left(u + mv_0\right)}{\rho\left(1+m^2\right)} + g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) \tag{14}$$

$$\frac{\partial v_0}{\partial t'} = \nu \frac{\partial^2 v_0}{\partial y^2} - 2\Omega' u + \frac{\sigma B_0^2 \left(mu - v_0\right)}{\rho\left(1+m^2\right)} \tag{15}$$

Here u is the axial velocity (along the direction of the plate) and v_0 is the transverse velocity (transverse to the main flow), the initial and boundary conditions are given by

$$\left.\begin{aligned} &u = 0, v_0 = 0, T' = T'_\infty, C' = C'_\infty, t' \le 0 \,\forall\, y \\ &u = u_0 \exp(a't'), v_0 = 0, t' > 0 \, at \; y = 0 \\ &T' = T'_w, C' = C'_w, t' > 0 \, at \; y = 0 \\ &u \to 0, v_0 \to 0, T' \to T'_\infty, C' \to C'_\infty \; as \; y \to \infty \end{aligned}\right\} \tag{16}$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^*\sigma\left(T'^4_\infty - T'^4\right) \tag{17}$$

It is assumed that temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \tag{18}$$

By using equations (17) and (18), equation (3) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 + 16a^*\sigma T'^3_\infty\left(T'_\infty - T'\right) \tag{19}$$

Let us introducing the following non-dimensional quantities

$$\left.\begin{aligned} &U = \frac{u}{u_0}, V = \frac{v_0}{u_0}, Y = \frac{yu_0}{\nu}, t = \frac{t'u_0^2}{\nu}, \Omega = \frac{\Omega'\nu}{u_0^2}, M^2 = \frac{\sigma B_0^2 \nu}{2\rho u_0^2} \\ &\theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \Pr = \frac{\nu\rho C_p}{k}, Sc = \frac{\nu}{D}, \\ &Gr = \frac{g\beta\nu\left(T'_w - T'_\infty\right)}{u_0^3}, R = \frac{16a^*\sigma\nu^2 T'^3_\infty}{ku_0^2}, a = \frac{a'\nu}{u_0^2} \\ &Gc = \frac{g\beta^*\nu\left(C'_w - C'_\infty\right)}{u_0^3}, \; Ec = \frac{u_0^2}{c_p\left(T'_w - T'_\infty\right)} \end{aligned}\right\} \tag{20}$$

Where the rotation parameter Ω, the thermal Grashof number Gr, the mass Grashof number Gc, the Prandtl number $\Pr$, the Schmidt number Sc, the radiation parameter R, the dimensionless time t, the accelerating parameter a, the dimensionless velocity u and v_0, Hall parameter m, Hartmann number M, Eckert number Ec, C is the dimensionless concentration and θ is the dimensionless temperature.

Making use of equation (20) equations (14), (15), (19) and (4), we get non-dimensional form as

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial Y^2} + 2\Omega V - \frac{2M^2\left(U + mV\right)}{1+m^2} + Gr\theta + GcC \tag{21}$$

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial Y^2} - 2\Omega U + \frac{2M^2\left(mU - V\right)}{1+m^2} \tag{22}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{\Pr}\theta + Ec\left(\frac{\partial U}{\partial Y}\right)^2 \tag{23}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \tag{24}$$

The boundary conditions for corresponding order are

$$\left.\begin{aligned} &U = 0, V = 0, \theta = 0, C = 0 \; at \; t \le 0 \; for \; all \; Y \\ &t > 0, U = \exp(a't'), V = 0, \theta = 1, C = 1 \; at \; Y = 0 \\ &U \to 0, V \to 0, \theta \to 0, C \to 0 \; as \; Y \to \infty \end{aligned}\right\} \tag{25}$$

3. Method of solution

To solve the set of partial differential equations (21-24), the finite element method has been used. The finite element method is powerful technique for solving ordinary and partial differential equations. This method is so general that it can be applied to a wide variety of engineering problems including heat transfer, fluid mechanics, solid mechanics and chemical processing. An excellent description of this method is presented in the text books by Bathe [18], Reddy [19], and Roland W. Lewis et al. [20]. The finite element method (FEM) consists of following steps.

3.1. Discretization of the domain

The fundamental concept of the FEM is to divide the region of the problem into small connected pieces, called finite elements. The group of elements is called the finite element mesh. These finite elements are associated in a non-overlapping manner, such that they completely cover the entire space of the problem.

3.2. Invention of the Element Equations

i) A representative element is selected from the mesh and the variationail formulation of the given problem is created over the typical element.

ii) Over an element, an approximate solution of the variation problem is invented, and by surrogating this in the system, the element equations are generated.

iii) The element matrix, which is also known as stiffness matrix, is erected by using the

element interpolation functions.

3.3. Assembly of the element equations

The algebraic equations so achieved are assembled by imposing the inter element continuity conditions. This yields a large number of mathematical equations known as the global finite element model, which governs the whole domain.

3.4. Imposition of the boundary conditions

On the accumulated equations, the initial and boundary conditions (25) are imposed.

3.5. Solution of assembled equations

The assembled equations so obtained can be solved by any of the numerical methods, namely, Gauss elimination technique, LU decomposition technique, and the final matrix equation can be solved by iterative technique. For computational purposes, the coordinate Y is varied from 0 to $Y_{\max} = 5.0$, where $Y_{\max}$ represents infinity $i.e.$, external to the momentum, energy and concentration edge layers.

In one-dimensional space, linear and quadratic elements, or element of higher order can be taken. The entire flow region is divided into 60 quadratic elements of equal size. Each element is three-nodded, and therefore the whole domain contains 180 nodes. At each node, four functions are to be evaluated; hence, after assembly of the element equations, we acquire a system of 720 equations which are nonlinear. Therefore, an iterative scheme must be developed in the solution. After striking the boundary conditions, a system of equations has been obtained which is solved mathematically by the Gauss elimination method while maintaining a correctness of 0.000001. A convergence criterion based on the relative difference between the present and preceding iterations is employed. When these differences satisfy the desired correctness, the solution is assumed to have been congregated and iterative process is terminated. The Gaussian quadrature is applied for solving the integrations.

The rate of heat transfer and mass transfer between the plate and the fluid, molecular concentration respectively. It is found by using the non-dimensional quantities, the Nusselt number Nu and the Sherwood number Sh. The Nusselt number is defined as negative gradient of the temperature. The Sherwood number is negative gradient of concentration.

Sherwood number $$Sh = -\left(\frac{\partial C}{\partial Y}\right)_{Y=0} \tag{26}$$

Nusselt number $$Nu = -\left(\frac{\partial \theta}{\partial Y}\right)_{Y=0} \tag{27}$$

4. Validation of numerical results

In the processes of validating the numerical result's accuracy by Mat lab based finite element code for Sherwood number and Nusselt number, the present results when Eckert number absent (i.e., $Ec = 0$) compared with the results obtained in earlier studies of Muthucumaraswamy and Prema (2016). It conforms that the present results are valid and agree to published literature and comparison warrant confidence in the finite element code which can be used in presenting further results quantitatively and graphically. From table I, Sherwood number Sh increases when the values of Sc increase at $t = 0.2$

but it decreases when the values of time increase at $Sc = 0.6$. From table II, Nusselt number decrease when the values of time t increase.

Table I: Comparison of Sherwood number Sh when $Ec = 0.0$

Sc	t	Previous Results (2016) (Sherwood number Sh)	Present Results (Sherwood number Sh)
0.3	0.2	0.6912	0.691257
0.16	0.2	0.5048	0.504843
0.78	0.2	1.1145	1.114572
0.6	0.2	0.9774	0.977463
2.01	0.2	1.789	1.78915
0.6	0.4	0.6912	0.691239
0.6	0.6	0.5644	0.564458

Table II: Comparison of Nusselt number Nu when $Ec = 0.0$

t	Previous Results (2016) (Nusselt number Nu)	Present Results (Nusselt number Nu)
0.2	3.09056	3.090547
0.4	2.94847	2.948528
0.6	2.84173	2.841849
0.8	2.76595	2.766142

5. Result and discussion

The effects of heat and mass transfer on an unsteady MHD flow with Hall current, heat source, and viscous dissipation have studied. The governing equations of the corresponding flow field solved by the finite element method. The effects of physical parameters on velocity components, temperature and concentration are analyzed and expressed in the figures 2-23. In this study following default parameter values of finite element computations are considered: $Gr = 5.0, Gm = 5.0, M = 0.5, m = 0.5, \Omega = 0.1,$ $\Pr = 0.71, R = 5.0, Sc = 0.6, Ec = 1.0, a = 0.1, t = 0.2$ All graphs correspond to these values unless specifically indicated on the appropriate graph.

Thermal Grashof number Gr is the ratio of buoyancy forces to the viscous forces. It signifies the relative magnitude of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. Figure 2 and 3 represents the effect of thermal Grashof number on primary and secondary velocities. They displays that the velocity begins to increase by increasing the values of thermal Grashof number Gr. This is due the reason that the thermal buoyancy force tends to accelerate fluid flow. The mass Grashof number Gc defines the ratio of the species buoyancy force to the viscous hydrodynamic force. Figure 4 and 5 depicts the effect of mass Grashof number Gc on primary and secondary velocities. From Figures it is noticed that there is an increase in fluid velocity in boundary layer region on increasing mass Grashof number Gc this happens due to the reason that concentration buoyancy force has a tendency to accelerate fluid velocity. The effect of the Hartmann number M is shown in figure 6 and 7. It is observed that decrease

in the primary velocity as the Hartmann number M increases, because the presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction, as in the present study. This resistive force slows down the fluid primary velocity component as shown in figure. But it has a different action on the other side in the case of the secondary velocity. Figure 8 and 9 express the influence of hall current m on the primary and secondary velocity profile. That is Hall current tends to accelerate primary fluid velocity in a region and Hall current tends to accelerate secondary fluid velocity throughout the region which is consistent with the fact that Hall current induces secondary flow in the flow-field. Figures 10 and 11 demonstrate the impact of rotation parameter on primary and secondary velocities. From figures, rotation parameter holds back primary velocity and enhances secondary velocity.

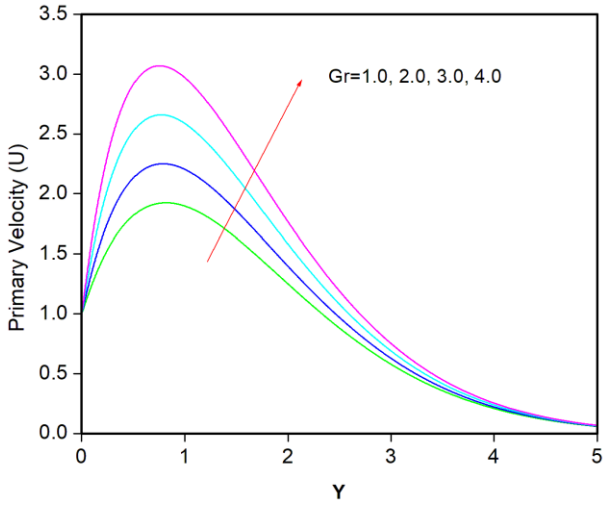

Figure 2. Influence of Gr on Primary Velocity profiles

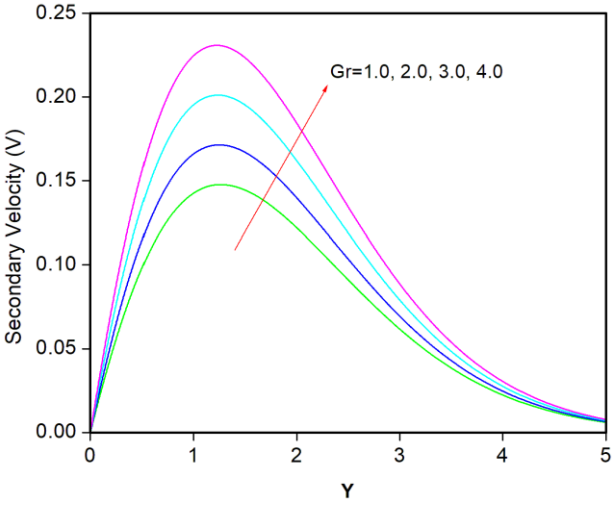

Figure 3. Influence of Gr on Secondary Velocity profile.

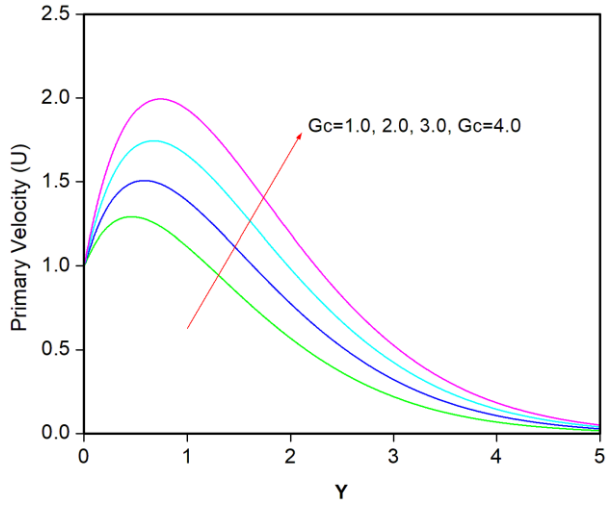

Figure 4. Influence of Gc on Primary Velocity profile.

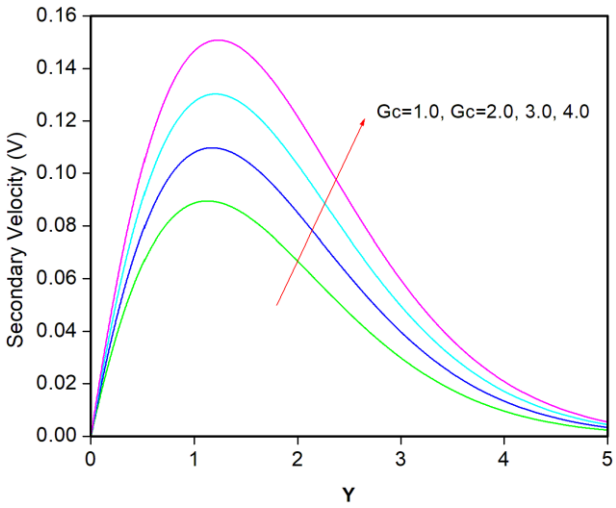

Figure 5. Influence of Gc on Secondary Velocity profile.

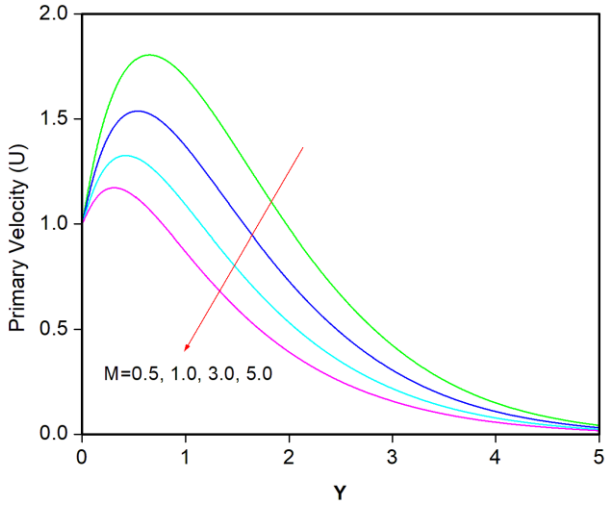

Figure 6. Influence of M on Primary Velocity profile.

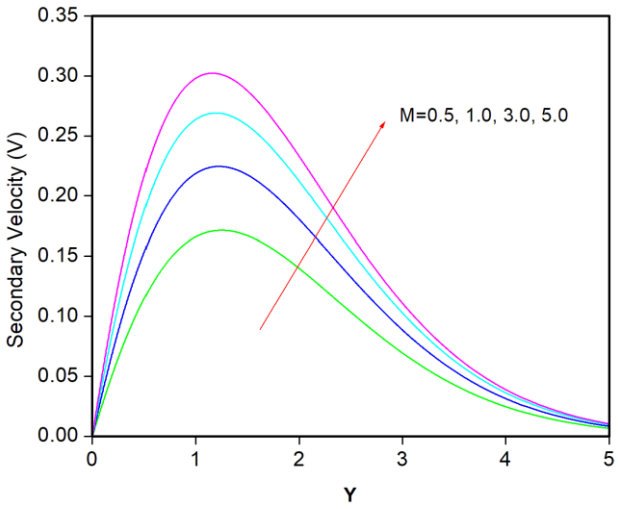

Figure 7. Influence of M on Secondary Velocity profile.

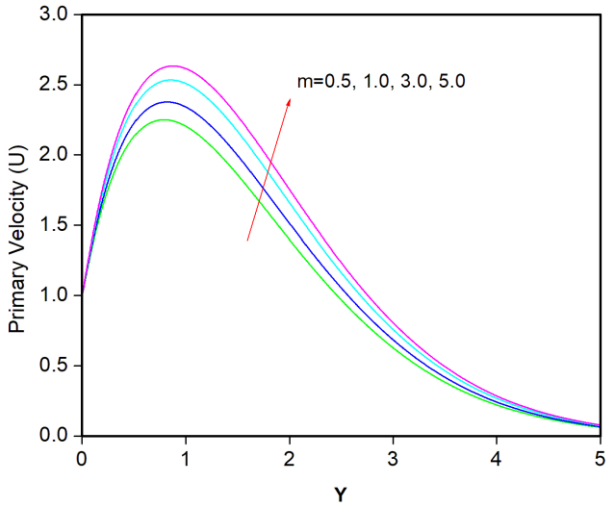

Figure 8. Influence of m on Primary Velocity profile.

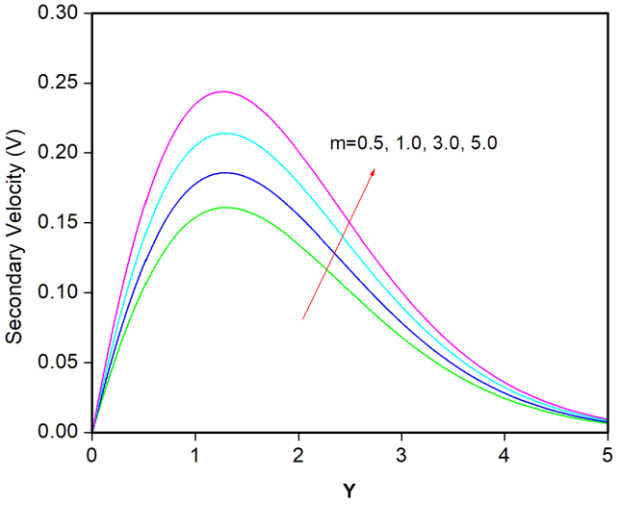

Figure 9. Influence of m on Secondary Velocity profile.

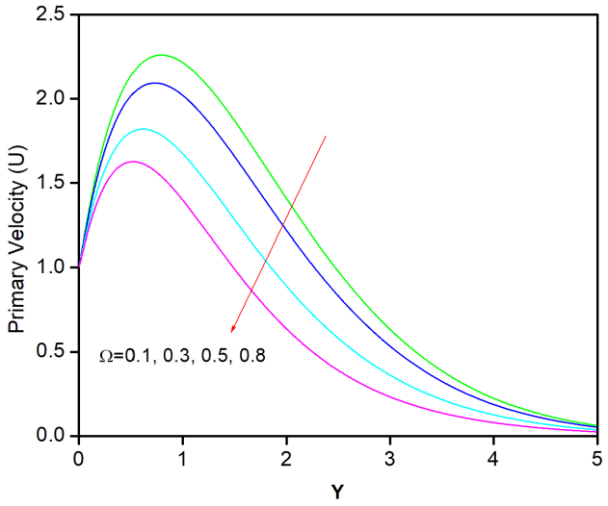

Figure 10. Influence of Ω on Primary Velocity profile.

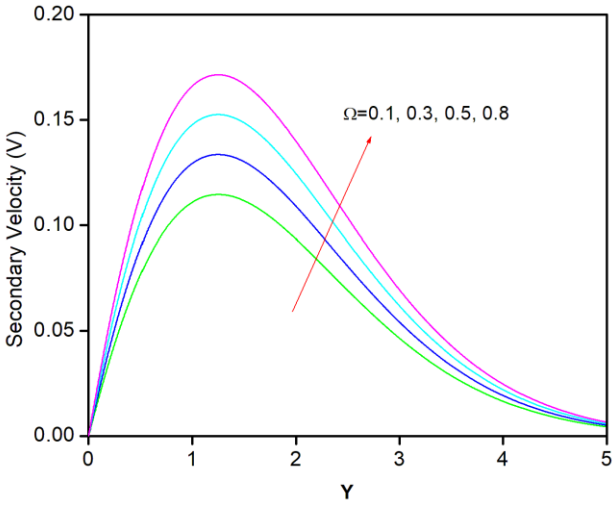

Figure 11. Influence of Ω Secondary Velocity profile.

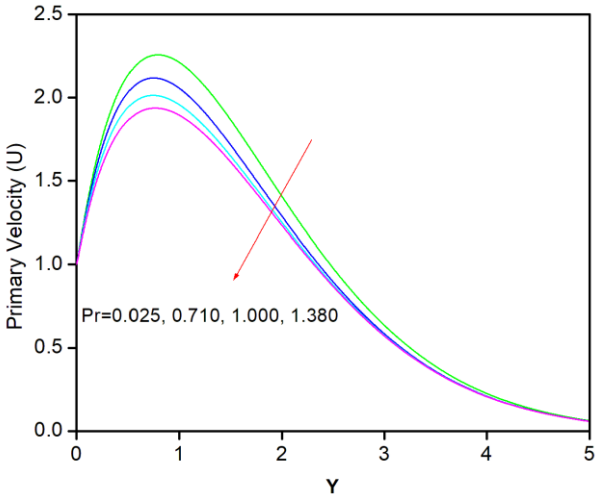

Figure 12. Influence of Pr on Primary Velocity profile.

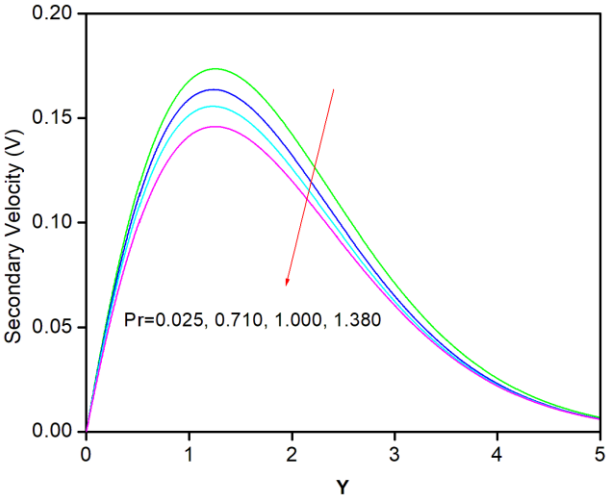

Figure 13. Influence of Pr on Secondary Velocity profile.

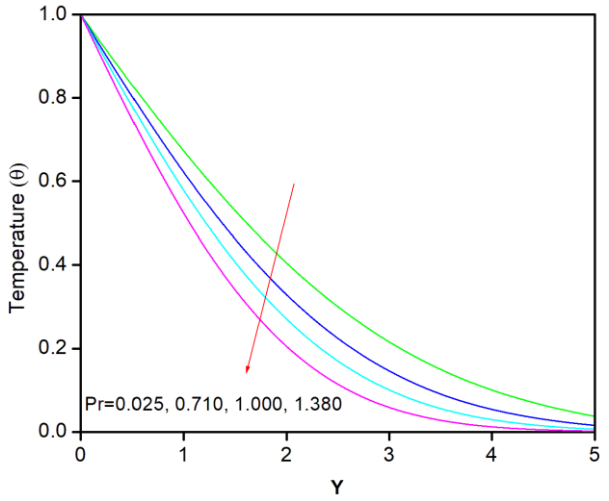

Figure 14. Influence of Pr on Temperature profile.

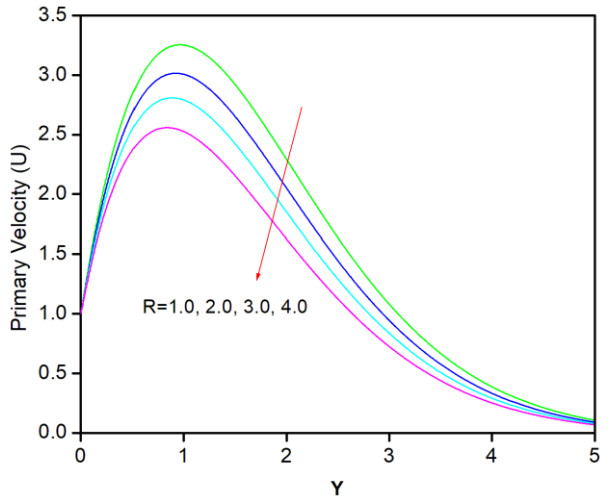

Figure 15. Influence of R on Primary Velocity profile.

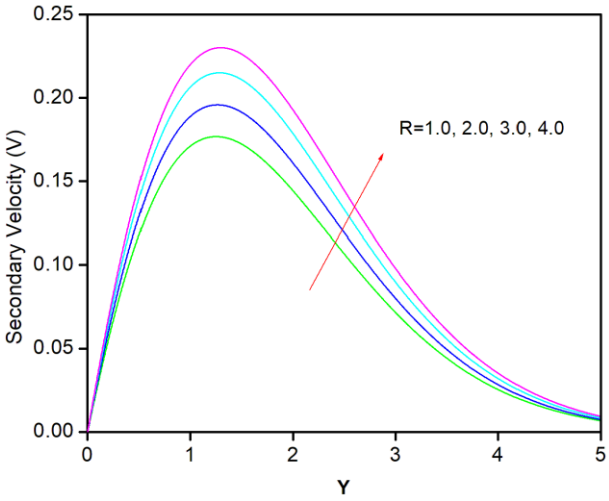

Figure 16. Influence of R on Secondary Velocity profile.

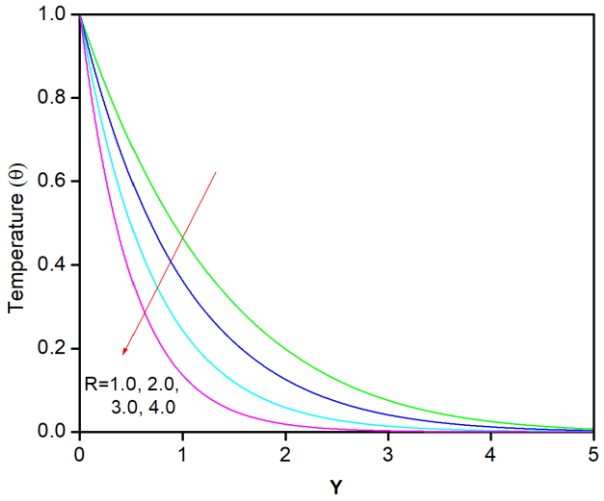

Figure 17. Influence of R on Temperature profile.

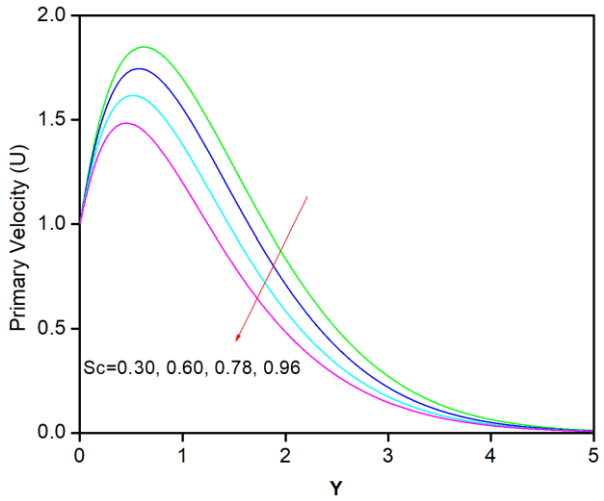

Figure 18. Influence of Sc on Primary Velocity profile.

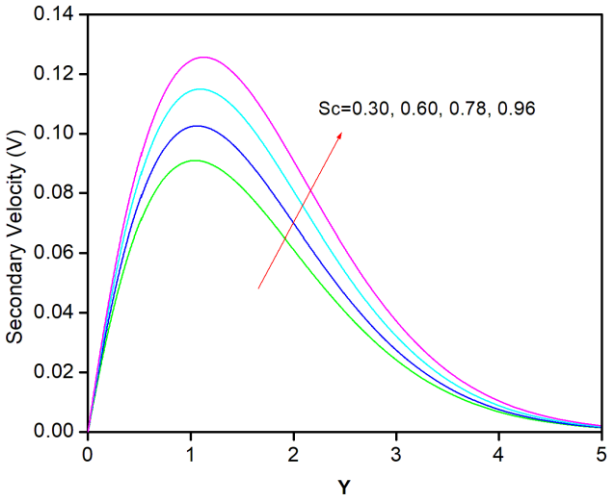

Figure 19. Influence of Sc on Secondary Velocity profile.

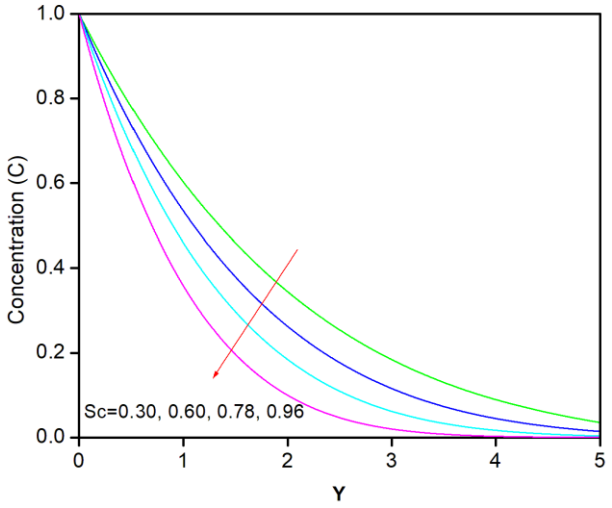

Figure 20. Influence of Sc on Concentration profile.

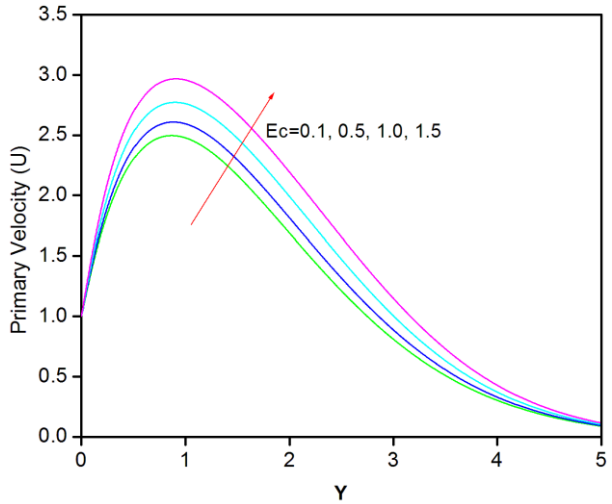

Figure 21. Influence of Ec on Primary Velocity profile.

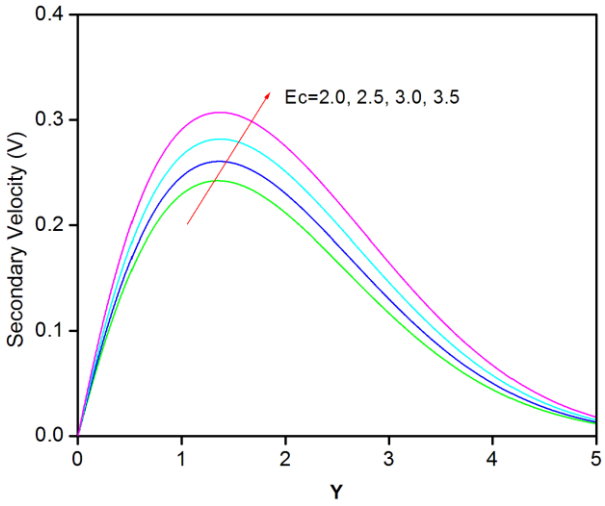

Figure 22. Influence of Ec on Secondary Velocity profile.

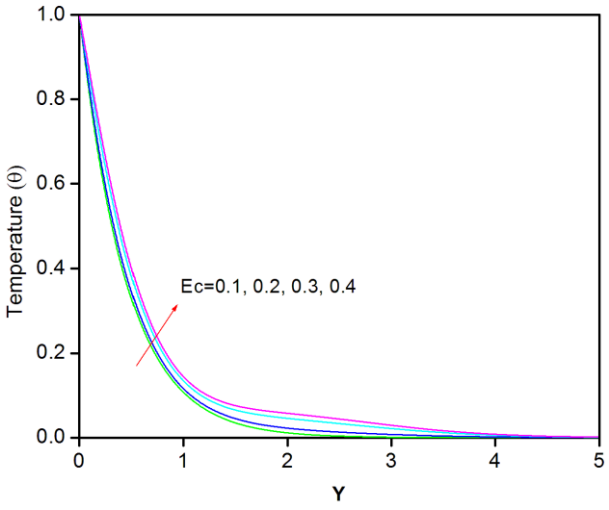

Figure 23. Influence of Ec on Temperature profile.

The Prandtl number is the ratio of Molecular diffusivity of momentum to molecular diffusivity of heat. From Figure 12, 13 and 14 the velocity components and temperature profiles decrease with an increase in dimensionless parameter Prandtl number because this is due to fact that the thermal boundary layer decreases with the increasing values of the Prandtl number. Primary velocity and temperature falls when radiation parameter R rise but secondary velocity increases with Radiation parameter R which is described in figure 15, 17 and 16 respectively. The Schmidt number represents the ratio of the momentum to the mass diffusivity. And it measures the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic and concentration boundary layers. By looking into the figure 18, 19 and 20 we observe that drop in primary velocity and concentration with an increase in Schmidt number and in secondary velocity increasing is generated with increasing Schmidt number. The Eckert number Ec expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. Although this parameter is often used in high speed compressible flow, for example in rocket aerodynamics at very altitude, it has significance in high temperature

incompressible flows, which are encountered in chemical engineering systems, radioactive waste repositories, nuclear engineering systems etc. When Ec is increased, it is seen from Figs. 21 and 22 that both the velocity components U and V increase, which is physically expected. Further, the effect of Eckert number Ec on the temperature is shown in Fig. 23. The effect of viscous dissipation on flow field is to increase the energy, yielding a greater fluid temperature and as a consequence the greater buoyancy force. The increase in the buoyancy force due to an increase in the dissipation parameter enhances the temperature.

6. Conclusions

Problem of viscous dissipation effects on transient magneto hydrodynamic flow past an exponentially accelerated infinite isothermal vertical plate with hall current is considered which is an extension of the work of Muthucumaraswamy and Prema (2016). Governed partial differential equations solved using FEM (Finite Element Method) subject to the initial and boundary conditions. Grid independence test conducted in detail. Validation of solutions with published literature is also included for several special cases, namely in the absence of viscous dissipation. The current computations are

- Primary velocity reduced whenever rotation parameter, radiation parameter, magnetic field parameter, Prandtl number, Schmidt number increased. It increased with increased thermal Grashof number, Solutal Grashof number, Hall parameter and viscous dissipation.
- Secondary velocity decreased when Prandtl number increase and it increased when thermal Grashof number, Solutal Grashof number, magnetic field, Hall parameter, rotation parameter, radiation parameter, Schmidt number and Eckert number are increase.
- Temperature decreased when Prandtl number and radiation parameter are increase, it increase with increasing Eckert number.
- Species concentration reduced when Schmidt number increase.

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