Modified One's approach to solve assignment problem

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ABSTRACT:

Assignment problem is an important problem in mathematics and is also discuss in real physical world. In, this paper we attempt to modify One's assignment method for solving assignment problem with algorithm and solution steps. We examine two numerical examples by using modified method and compute by Hungarian method. Also, we compare the optimal solutions among this modified method and Hungarian method. The modified method is a systematic procedure, easy to apply for solving assignment problem.

Keywords: Assignment problem, Hungarian assignment method (HA-method), One's method, Cost minimization, Optimization.

INTRODUCTION:

The assignment method helps in picking the "Task" and matching with the most appropriate "Agent", which can give most efficient outcome. It was first sensed by Votaw and Ordenin [1] in 1952. Application of the assignment model are numerous and widespread in our daily life. It is not just limited to assignment jobs to employees or machines; its scope is widespread. Presently, in the literature, some of the classical algorithms developed for AP solution are: Branch Boundary Algorithm, Brute Force Algorithm, and Hungarian Algorithm. There are also heuristic algorithms (genetic algorithms, parallel auction algorithm, penalty method, and greedy method) for solving this problem. Among these, the Greedy algorithm is one of the most preferred methods because it is easy to apply and requires a small number of iterations. There are more than 10 algorithms that are similar in performances (and which one is the best depends on specific situations) in these days, and studies about this topic are still on-going [1]. Classical and heuristic algorithms for assignment problems are given in Table 1.

Table 1: Algorithms for assignment problems

Classical Algorithms	**Heuristic Algorithms**
Simplex Method (LP)	OACE Algorithm
Transportation Method	Auction Algorithm
Brute Force Algorithm	Greedy Algorithm
The Branch Boundary Algorithm	Genetic Algorithm
Hungarian Algorithm	Harmony Search Algorithm
Lagrange Relaxation Algorithm	Penalty Method

Among diverse approaches developed for assignment problem, Hungarian method [2,3], linear approaches [4], and the newer one- Auction algorithm [5] and heuristic algorithms are commonly used. The Branch Boundary Algorithm, reported by Breu and Burdet (1974), Wolsey and Nemhauser (1988), and Rijavec (1992) is the only known algorithm for solving the multidimensional assignment problem optimally [6, 7, 8]. For problems of any size, since the time required for computing the optimal solution grows exponentially with the size of the problem, this algorithm is impractical [9]. Since assignment problem in Operations Research is a main problem, many studies have been conducted about this topic and various algorithms have emerged [10]. Basirzadeh [12] worked and developed a Hungarian like method knows as Ones Assignment method which maximizes or minimizes the objective function. Muley and Ghadle [13] presented a new step in Basirzadeh's method and hence named it revised Ones assignment method. H. Gamel [14] also brought to light the drawbacks of One assignment method with some modification. Dil Afroz, and Anwar Hossen [15] proposed a new method to solve assignment problem. Although these algorithms [12-14] are efficient in general, they possess drawbacks which can lead to poor performance. In this study, modified version of the Ones Assignment method will be present to attain an exact optimal solution.

Mathematical formulation of an Assignment Problem:

Suppose there are n-jobs for a factory and has n-machines to process those jobs. A job i (i = 1,2, 3, ……., n) when processed by machine j (j =1,2,3,….., n) is assumed to incur a cost c_{ij} . The assignment is to be made in such a way that each job can be associated with one and only one machine. Determine an assignment of jobs to machines to minimize the overall cost.

Suppose x_{ij} is a variable, which is defined as:

$$x_{ij} = \begin{cases} 1, & if\ the\ i^{th}\ job\ is\ assign\ to\ the\ j^{th}\ machine \\ 0, & if\ the\ i^{th}\ job\ is\ not\ assign\ to\ j^{th}\ machine \end{cases}$$

Now as the problem forms one to one basis or one job is to be assigned to one facility or machine. Therefore

$\sum_{i=1}^{n} x_{ij} = 1$ and $\sum_{j=1}^{n} x_{ij} = 1$

The total assignment cost will be given by

$U = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$

The above definition can be developed into mathematical model as follows:

Determine $x_{ij} > 0$, (i, j = 1,2, 3…n) to Minimize $U = \sum_{1=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$

Subjected to constraints

$\sum_{i=1}^{n} x_{ij} = 1$, when i = 1,2,3…… n, $\sum_{j=1}^{n} x_{ij} = 1$, when j = 1,2,3 …….n

and x_{ij} is either zero or one.

THEOREM 1:

If a constant is added or subtracted to every element of a row or column of the cost matrix of an assignment problem, the resulting assignment problems has the same optimal solution as the original problem and vice versa.

Alternatively, if $x_{ij} = x_{ij}^*$

Z = $\sum_{1=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$ over all x_{ij} such that

$\sum_{i=1}^{n} x_{ij} = \sum_{j=1}^{n} x_{ij} = 1 >= 0$ then $x_{ij} = x_{ij}^*$ also $Z^* = \sum_{1=1}^{n}\sum_{j=1}^{n} c_{ij}^* x_{ij}$

Where $c_{ij}^* = c_{ij} + u_i + v_j$ for all I, j = 1,2,3….where u_i, v_j are real numbers.

Proof:

also $Z^* = \sum_{1=1}^{n}\sum_{j=1}^{n} c_{ij}^* x_{ij} = \sum_{1=1}^{n}\sum_{j=1}^{n} (c_{ij} \pm u_i \pm v_j)x_{ij}$

$= \sum_{1=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} \pm \sum_{i=1}^{n} u_i \sum_{j=1}^{n} x_{ij} \pm \sum_{i=1}^{n} x_{ij} \sum_{j=1}^{n} v_j$

$= \sum_{1=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} \pm \sum_{i=1}^{n} u_i \pm \sum_{j=1}^{n} v_j$, since $\sum_{i=1}^{n} x_{ij} = \sum_{j=1}^{n} x_{ij} = 1$

$Z^* = Z \pm \sum_{i=1}^{n} u_i \pm \sum_{j=1}^{n} v_j$.

This shows that the minimization of the new objective function Z* yields the same solution as the minimization of original objective function Z, because $\sum u_i$ and $\sum v_j$ are independent of x_{ij} .

Theorem 2:

If any rows of a cost matrix are divided by a constant $\alpha \neq 0$ than the solution to the assignment cost matrix still exists.

Proof:

Suppose that row r of the cost matrix is divided by α,

where $\alpha = \min\{c_{rj}\}$, j = 1, 2, K, n; $c_{rj} \neq 0$ for all j,

$$Z^* = \sum_{1=1}^{n}\sum_{j=1}^{n} c_{ij}^* x_{ij} = \sum_{\substack{1=1 \\ i\neq r}}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{j=1}^{n} (c_{rj}/\alpha)x_{rj} = \sum_{\substack{1=1 \\ i\neq r}}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} + (1/\alpha)\sum_{j=1}^{n} c_{rj}x_{rj} < Z$$

We have shown that Z*< Z, that is, we have shown the existence of the solution if the entries of a row of the cost matrix are divided by α.

The algorithm is as follows:

Step 1: Find the smallest number (cost) of each row. Subtract this smallest number from every number in that row. If the minimum element is 1 then directly go to **step 2.**

Step 2:
Now add 1 to all element and we get at least one 1 in each row, in case if the problem is unbalanced, we make it balanced by adding a dummy row or column with '1' as entries. then make assignment in terms of ones.

Note: If there are more than one ones in any column that can be assigned than make assignment to the cell which has minimum cost in the original cost matrix and if there is a tie between the cost of such cells than find the difference of the minimum and next minimum cost and make assignment to the row which has maximum difference and if there is a tie between the difference than find the difference of the minimum and third minimum value and make assignment to the row which has maximum difference.
If there are some rows and columns without assignment, then we cannot get the optimum solution. Then we go to the next step.

Step 3: Draw the minimum number of lines passing through all ones by using the following procedure:

i. Mark (√) rows that do not have assignments.
ii. Mark (√) columns that have crossed ones in that marked rows.
iii. Mark (√) rows that have assignments in marked columns.
iv. Repeat (ii) and (iii) till no more rows or columns can be marked.
v. Draw straight lines through all unmarked rows and marked columns.

If the number of lines drawn is equal to the number of rows or columns, then the current solution is optimal solution. Otherwise go to next step.

Step 4: Select the smallest number of the reduced matrix not covered by the lines. Divide all uncovered numbers by this smallest number. Other numbers covered by lines remain unchanged. Then we get some new ones in row and column. Again, make assignment in terms of ones.

Step 5: If we cannot get the optimal assignment in each row and column. Then repeat steps (3), (4) and successively till an optimum solution is obtained.

Solve the following balanced assignment problem using Proposed Method.

Consider the problem of assigning five jobs to five persons. The assignment costs are given below.

	1	***2***	***3***	***4***	***5***
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

Determine the optimum assignment schedule and minimum assignment cost.

Step 1: Find the minimum element of each row and subtract it from each element in that row. Then the reduced matrix is as follows:

	1	***2***	***3***	***4***	***5***
A	7	3	1	5	0
B	0	9	5	5	4
C	1	6	7	0	4
D	4	3	1	0	3
E	4	0	3	4	0

Step 2: Now add 1 to all element.

	1	***2***	***3***	***4***	***5***
A	8	4	2	6	1
B	1	10	6	6	5
C	2	7	8	1	5
D	5	4	2	1	4
E	5	1	4	5	1

Step 3: Now make initial assignment. In 4th column we have two 1's so using step 2 we would assign in D row as in original cost matrix 0 in D row and 2 in C row.

	1	***2***	***3***	***4***	***5***
A	~~8~~	~~4~~	~~2~~	~~6~~	1
B	~~1~~	~~10~~	~~6~~	~~6~~	~~5~~
C	2	7	8	1	5
D	5	4	2	~~1~~	4
E	5	~~1~~	~~4~~	~~5~~	~~1~~

Here, C row and 3rd column do not have any assignment. Thus, the solution is not optimum, and we go to next.

Step 4: Select the smallest number (2) of the reduced matrix not covered by the lines. Divide all uncovered number by this smallest number. Other numbers covered by lines remain unchanged. Then make assignment again.

	1	***2***	***3***	***4***	***5***
A	8	4	2	6	1
D	1	10	6	6	5
C	~~1~~	3.5	4	1	2.5
D	2.5	2	1	~~1~~	2
E	5	1	4	5	~~1~~

Here we see that all ones are either assigned or crossed out. That is, the total assigned one's is 5 which is equal to the number of rows or columns. And the solution is (1,5), (2,1), (3,4), (4,3), (5,2). So, the total cost = 1 + 0 + 2 + 1 + 5 =9

Solve the Above assignment problem using Hungarian Assignment Method.
Solution:

Step 1: Select the minimum element in each row and subtract this element from every element in that row.

	1	***2***	***3***	***4***	***5***
A	7	3	1	5	0
B	0	9	5	5	4
C	1	6	7	0	3
D	4	3	1	0	3
E	4	0	3	4	0

Step 2: Select the minimum element in each column and subtract this element from every element in that column.

	1	***2***	***3***	***4***	***5***
A	7	3	0	5	0
B	0	9	4	5	4
C	1	6	6	0	3
D	4	3	0	0	3
E	4	0	2	4	0

Step 3: Make initial assignment.

	1	***2***	***3***	***4***	***5***
A	7	3	~~0~~	5	0
B	0	9	4	5	4
C	1	6	6	0	3
D	4	3	0	~~0~~	3
E	4	0	2	4	~~0~~

Here we see that all zeros are either assigned or crossed out. That is, the total assigned zero's is 5 which is equal to the number of rows or columns. And the solution is (1,5), (2,1), (3,4), (4,3), (5,2). And the total cost= 1 + 0 + 2 + 1 + 5 = 09.

Solve the following Unbalanced assignment problem using modified Method.

A batch of 4 jobs can be assigned to 5 different machines. The set-up time for each job on various machines is given below.

	1	***2***	***3***	***4***	***5***
A	11	10	4	2	8
B	7	11	10	14	12
C	5	6	9	12	14
D	13	15	11	10	7

Find the minimum time that the work can be completed.

Solution:

Step 1: Find the minimum element of each row and subtract it from each element in that row. Then the reduced matrix is as follows:

	1	***2***	***3***	***4***	***5***
A	8	9	2	0	6
B	0	4	3	7	5
C	0	1	4	7	9
D	6	8	4	3	0

Now add 1 to all the elements and make it balanced by adding a dummy row with '1' as entries.

	1	***2***	***3***	***4***	***5***
A	9	10	3	1	7
B	1	5	4	8	6
C	1	2	5	8	10
D	7	9	5	4	1
E	1	1	1	1	1

Now make assignment in terms of 1.

	1	***2***	***3***	***4***	***5***
A	~~9~~	~~10~~	~~3~~	~~1~~	~~7~~
B	1	5	4	8	6
C	~~1~~	2	5	8	10
D	~~7~~	~~9~~	~~5~~	~~4~~	~~1~~
E	~~1~~	~~1~~	1	~~1~~	~~1~~

Here we have no assignments in 3rd row and 2nd column, so we go to step 4. Since 2 is the smallest of the uncovered element we divide the uncovered by 2 and get.

	1	***2***	***3***	***4***	***5***
A	9	10	3	1	7
B	1	5/2	2	4	3
C	~~1~~	1	5/2	4	5
D	7	9	5	4	1
E	~~1~~	~~1~~	1	~~1~~	~~1~~

Here we see that all ones are either assigned or crossed out. That is, the total assigned one's is 4 which is equal to the number of rows or columns. And the last row is dummy, so we do not consider it for the solution and hence the solution is (1,4), (2,1), (3,2), (4,5). So, the total cost = 2+7+6+7= 22,

Also, the total cost by Hungarian method is 22.

Conclusion:

In this study modified version of One's assignment method is introduced for solving assignment problem. The original method has some drawbacks at the step of making assignments and that was only applicable to balanced case. This modification presented in this paper overcomes those drawbacks and is also applicable to unbalanced case. Despite having a few new features and more steps the new modified Ones assignment method is precise, efficient and is equally suitable for different types of assignment problems

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